

FOR AND AGAINST METAPHYSICS IN THE MODAL INTERPRETATION OF QUANTUM MECHANICS

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ABSTRACT

In this paper we discuss the relation of quantum theory to the problem of metaphysics. Based on metaphysical and anti-metaphysical stances, we put forward an ‘interpretational map’ of quantum mechanics in general and of the modal interpretation in particular. Thus, within the modal interpretation, we distinguish between: *Modal Interpretations (which start from) the Mathematical Formalism* (MIMF) and *Modal Interpretations (which start from) Metaphysical Principles* (MIMP). Finally, we argue for a middle path in between metaphysical principles and the formal conditions imposed on quantum mechanics.

1 Introduction: For or Against Metaphysics?

The term “metaphysics” comprises a series of many different definitions such as: ‘first philosophy’, ‘the study of Being qua Being’, ‘study of the foundations of Being’. Metaphysics occupies itself with questions which are beyond physical experience. For some, it can be considered as a supreme form of knowledge, while for others, it remains a disgusting occupation constituted by unfruitful discussions. Already in the mid 19th century the criticism to metaphysics had appeared explicitly in the

positivistic philosophy of the French Auguste Comte and the British John Stuart Mill. Positivism had derived from Enlightenment thinkers like Pierre-Simon Laplace and many others, but was firstly systematically theorized by Comte, who saw the scientific method as replacing metaphysics in the history of thought. Positivism is a philosophy which states that the only authentic knowledge is knowledge that is based on actual sense experience. Such knowledge can come only from affirmation of theories through strict scientific method. Metaphysical speculation is avoided. Ernst Mach is maybe one of the most influential positivistic thinkers of the 19th century. Mach, a physicist himself, was primarily interested in the nature of physical knowledge. His investigations led him to the conclusion that science is nothing but the systematic and synoptical recording of data of experience. In close analogy to Darwinistic ideas Mach conceived the evolution of knowledge in physical theories as a process of “struggle for life” and “survival of the fittest”. In his *Analysis of Sensations* (Mach, 1959), Mach concluded that primary sensations constitute the ultimate building blocks of science, inferring at the same time that scientific concepts are only admissible if they can be defined in terms of sensations. Although Mach had been himself a neo-Kantian, within his neo-positivist conception of science, he stated that we should reject every a priori element in the constitution of our knowledge about things. Science would be then nothing but a conceptual reflection of the facts which are provided by sensations. Scientific propositions should be empirically verifiable and as a consequence, within this doctrine there is no place left for absolute concepts – as for example space and time.

Mach’s positivism played a significant role within the scientific revolutions – namely, relativity theory and quantum mechanics – that took place at the beginning of the 20th century. After some centuries, the categories and forms of intuition had become that which Kant had strived to attack in the metaphysics of his time, dogmatic and unquestionable elements. Kant had fought 17th century metaphysics, which meant for him the possibility to go beyond dogma and belief, to understand the

finite access with which every human being is confronted. By understanding the limits of human knowledge metaphysics would finally follow the secure path of science and show how (scientific) knowledge is possible. But his own philosophy had turned itself into new dogma. As noted by van Fraassen (2002, p. 2): “Kant exposed the illusions of Reason, the way in which reason overreaches itself in traditional metaphysics, and the limits of what can be achieved within the limits of reason alone. But on one hand Kant’s arguments were not faultless, and on the other there was a positive part to Kant’s project that, in his successors, engaged a new metaphysics. About a century later the widespread rebellions against the Idealist tradition expressed the complaint that Reason had returned to its cherished Illusions, if perhaps in different ways.”

The incisive criticism of Mach to the lack of foundation of the physical concepts in the theories of his time allowed a complete reformulation of the meaning and applicability of physical concepts. The importance of Mach’s positivistic ideas regards mainly the deconstruction of Kant’s *a priori* structure of thought. His analysis opened small cracks in the basic physical presuppositions connected with the metaphysics of his time, and so, prepared a period where those who followed were able to go beyond the impositions of classical physics. Only after Mach and his criticism to the *a priori*; the concepts of ‘space’, ‘time’, ‘substance’, ‘causality’, etc. could be discussed and deconstructed one by one. For example, as noticed by Bohr (Wheeler and Zurek, 1983, p. 106) himself, Heisenberg succeeded “in emancipated himself completely from the classical concept of motion by replacing from the very start the ordinary kinematical and mechanical quantities by symbols which refer directly to the individual processes demanded by the quantum postulate.” The Machian epistemological principle had broken the chains of the Kantian *a priori*s. A new experience was disclosed, a new region of thought had been created.

“In many respects the present appears as a time of insecurity of the fundamentals, of shaky foundations. Even the development of the exact sciences has not entirely escaped this mood of insecurity, as appears, for instance, in the phrases ‘crisis in the foundations’ in mathematics, or ‘revolution in our picture of the universe’ in physics. Indeed many concepts apparently derived directly from intuitive forms borrowed from sense-perceptions, formerly taken as matters of course or trivial or directly obvious, appear to the modern physicist to be of limited applicability. The modern physicist regards with scepticism philosophical systems which, while imagining that they have definitively recognized the *a priori* conditions of human understanding itself, have in fact succeeded only in setting up the *a priori* conditions of the systems of mathematics and the exact sciences of a particular epoch.” (Pauli, 1994, p. 95)

Albert Einstein, Werner Heisenberg and Pauli were close followers of Mach. This fact can be witnessed in Einstein’s interpretation of the photoelectric effect and Heisenberg’s interpretation of the cloud chamber. However, the problem which all these thinkers confronted was still that of physical reality. Thus, it is not strange to find out that the development of positivistic ideas, in the context of the Vienna circle, was criticized by all three of them on many occasions.

As explicitly remarked in its manifesto (Carnap, Hahn and Neurath, 1929), the Vienna Circle is characterized “essentially by two features. First, it is empiricist and positivist: there is knowledge only from experience [...] Second, the scientific world-conception is marked by the application of a certain method, namely logical analysis.” Following Frege, Russell and Carnap, logical analysis is the method of clarification of philosophical problems and the task of philosophy lies in the clarification of problems and assertions.¹ Logical analysis shows that there are two different kinds of statements; one kind includes statements

¹ For a more profound analysis of the development of the Vienna Circle see (Zuppone, 2010).

reducible to simpler statements about the empirically given; the other kind includes statements which cannot be reduced to statements about experience and thus they are devoid of meaning. Metaphysical statements belong to this second kind and therefore they are meaningless. Regarding these anti-metaphysical elements, Einstein remained at a distance from logical positivism and the Vienna Circle. As noted by Howard:

“Einstein was dismayed by the Vienna Circle’s ever more stridently anti-metaphysical doctrine. The group dismissed as metaphysical any element of theory whose connection to experience could not be demonstrated clearly enough. But Einstein’s disagreement with the Vienna Circle went deeper. It involved fundamental questions about the empirical interpretation and testing of theories.” (Howard, 2007, p. 73)

For Einstein (Wheeler and Zurek, 1983, p. vii), the starting point for physics was also a metaphysical stance: “Out yonder there was this huge world, which exists independently of us human beings and which stands before us like a great, eternal riddle, at least partially accessible to our inspection.” According to him, the guiding line of physics was to be described in the following terms:

“[...] it is the purpose of theoretical physics to achieve understanding of physical reality which exists independently of the observer, and for which the distinction between ‘direct observable’ and ‘not directly observable’ has no ontological significance; this aim furnishes the physicist at least part of the motivation for his work; but the only decisive factor for the question whether or not to accept a particular physical theory is its empirical success.” (A. Einstein quoted from Dieks, 1988, p. 175)

As noticed by Vassilios Karakostas (2004, p. 15): “[...] the concept of mind-independent reality is not strictly scientific; it is metaphysical by nature. It concerns the existence of things in themselves, absolutely independent of any act of perception or observation. Hence, it does not apply to empirical science proper because, by definition, it excludes the

empirical testing of its existence. It may be viewed, however, as a regulative principle in physics research, as a conviction which gives direction and motive to the scientific quest.” Heisenberg, also took positivism as developed by the Vienna Circle to be a definite aim of attack. In his autobiography, he writes:

“Positivist insistence on conceptual clarity is, of course, something I fully endorse, but their prohibition of any discussion of the wider issues, simply because we lack clear-cut enough concepts in this realm, does not seem very useful to me – this same ban would prevent our understanding of quantum theory.” (Heisenberg, 1971, p. 208)

And continues later on:

“The positivists have a simple solution: the world must be divided into that which we can say clearly and the rest, which we had better pass over in silence. But can anyone conceive of a more pointless philosophy, seeing that what we can say clearly amounts to next to nothing? If we omitted all that is unclear, we would probably be left with completely uninteresting and trivial tautologies.” (Heisenberg, 1971, p. 213)

For both Heisenberg and Pauli, the position they had against positivism was directly related to the denial of the “problem of reality”, the metaphysical question as related to a possible interpretation of quantum mechanics (QM). Pauli was maybe the most radical thinker of the quantum revolution and was ready to leave aside the Kantian *a priori* preconditions of understanding and replace them by new – still to be developed – concepts. As explicitly expressed by him, the crisis to which 20th century physics and philosophy confronts us – against the Kantian claim and its very different proponents –, relates to a proper development of the meaning of reality itself. In his own terms:

“When the layman says “reality” he usually thinks that he is speaking about something which is self-evidently known; while to me it appears to be specifically the most important and extremely difficult task of our time

to work on the elaboration of a new idea of reality.” (Pauli quoted from Laurikainen, 1998, p. 193)

The tension present in this debate can be related to a possible characterization of the history of Western thought as a confrontation between two main forces. On the one side the metaphysical or ontological force, which seeks to answer the most important questions of all, the question of Being qua Being; and on the other side, an anti-metaphysical or epistemological force, which seeks to understand the limits and constrains of such question. Analytic philosophy has been clearly, not only from an historical perspective but also methodologically, part of this second force.

Empiricism and logicism are two of the main sources of the origin of analytic philosophy. The central idea of positivism is that science should use theories as an instrument and should renounce to seek for explanation. The search for such explanations is a metaphysical enterprise, and as such, nothing but nonsense. As noticed by van Fraassen (2002, p. xviii) “Empiricist philosophers have always concentrated on epistemology, the study of knowledge, belief, and opinion, with a distinct tendency to advocate the importance of opinion.” Against the ontological concerns of the metaphysicians, analytic philosophers engaged in epistemological issues. Within analytic philosophy epistemology seems to remain the only sensible concern. Escaping from the true statements of the metaphysicians, of *episteme*, analytic philosophy remained closer to opinion and *doxa*. True knowledge was regarded with suspicion, as a dogma of the past, as a metaphysical idol with no proper fundament. According to van Fraassen (2002, p. 36), the history of analytic philosophy is also directly connected to a critic, a reaction against the metaphysical attitude of the continent in the 17th century. “The story of empiricism is a story of recurrent rebellion against a certain systematizing and theorizing tendency in philosophy: a recurrent rebellion against the metaphysicians.” However, and although analytic philosophy started from a revolution against metaphysics, the introduction of metaphysical

questions reappeared very soon within analytic philosophy itself. According to van Fraassen the rebellion against 17th century metaphysics was exactly what gave birth to analytic philosophy as such:

“As I see it, analytic philosophy – which is the strand to which I belong – began with a revolution that was subverted by reactionary forces. I am speaking here of reversion to a seventeenth century style of metaphysics. I do not reject all metaphysics, but this reversion I see as disastrous. Paradoxically, this disaster seems to be worst in two areas that scarcely relate to each other at all. I mean, on one hand, the area loosely characterized as “science and religion” studies and, on the other, academic analytic philosophy. Both suffer from unacknowledged as well as explicit metaphysics.” (van Fraassen, 2002, p. xviii)

As noticed by van Fraassen one of the most interesting and subversive starting points of analytic philosophy was very soon turned upsidedown.

“[...] with the rise of analytic philosophy something paradoxical happened. This movement began in a series of revolts, across Europe and America, against all forms of metaphysics. And lo, even before mid-century, some of its ablest adherents began to make the world safe for metaphysics again. Since then we have seen the growth of analytic ontology, analytic metaphysics, and it thrives today. Or so it seems. I say that metaphysics is dead. What I see is false consciousness, a philosophy that has genuinely advanced beyond the past, but a philosophy that misunderstands itself.” (van Fraassen, 2002, pp. 3-4)

After the second world war the philosophical analysis of science, and of quantum theory in particular, has been an almost exclusive field owned by analytic philosophy. As we have discussed above, this tradition inherits a deep criticism to metaphysics. However, the return to metaphysics within such same regions of thought seems to be a recursive element also in the analysis of physics in general and of the interpretation of QM in particular. In the following we shall concentrate in what we consider to be a particular example of the fight in between metaphysical

and anti-metaphysical positions within the philosophy of QM: modal interpretations.

2 The Modal Interpretation of Quantum Mechanics

The study of the modal character of QM was explicitly formalized in the seventies and eighties by a group of physicists and philosophers of science. Bas van Fraassen was the first one to formally include the reasoning of modal logic in QM. He presented a modal interpretation (MI) of quantum logic in terms of its semantical analysis (van Fraassen, 1973; 1981) which had the purpose to clarify which properties among those of the complete set structured in the lattice of subspaces of Hilbert space pertain to the system. Van Fraassen's position can be closely related, as he does himself, to Bohr's interpretation. Some years later, within the modal scheme, Simon Kochen presented an interpretation which related closely (Kochen, 1981) to the discussions between the founding fathers of the theory. Carl Friedrich von Weizsäcker and Theodor Görnitz (1987, p. 357) referred specifically to it in a paper entitled *Remarks on S. Kochen's Interpretation of Quantum Mechanics*. "We consider it is an illuminating clarification of the mathematical structure of the theory, especially apt to describe the measuring process. We would, however feel that it means not an alternative but a continuation to the Copenhagen interpretation (Bohr and, to some extent, Heisenberg)." Finally, Dennis Dieks also considered his own interpretation as a formalization of Bohr's ideas.² However, in his first papers Dieks went further in relation to the metaphysical presuppositions involved in Bohr's interpretation, making explicit that his own version of the MI (Dieks, 1988; 1989; 2005; see also: Dickson and Dieks, 2002) attempted to provide a realistic account "in terms of properties possessed

² Private discussion, Utrecht, December 2009.

by physical systems, independently of consciousness and measurements (in the sense of human interventions)” (Dieks, 2007). Later on, new versions of the MI, in line with the hidden variable program, were developed by Clifton, Bub, Dickson and Bacciagaluppi. The starting points and agenda of these authors take a different line of research and development of modal ideas. In the following we shall review these very different attempts.

As noted by Dirac in the first chapter of his famous book, the existence of superpositions is responsible for the striking difference of quantum behavior from the classical one. In fact, the photon being in a superposition of states must be accepted if we want to explain interference effects (Dirac, 1974). Superpositions are also central when dealing with the measurement process, where the various terms associated with the possible outcomes of a measurement must be assumed to be present together in the description. This fact leads van Fraassen to the distinction between value-attributing propositions and state-attributing propositions, between *value-states* and *dynamic-states*:

“[...] a *state*, which is in the scope of quantum mechanics, gives us only probabilities for actual occurrence of events which are outside that scope. They can't be entirely outside the scope, since the events are surely described if they are assigned probabilities; but at least they are not the same things as the states which assign the probability.

In other words, the state delimits what can and cannot occur, and how likely it is – it delimits possibility, impossibility, and probability of occurrence – but does not say what actually occurs.” (van Fraassen, 1991, p. 279)

So van Fraassen distinguishes propositions about events and propositions about states. Propositions about events are value-attributing propositions $\langle \mathbf{A}, \sigma \rangle$, they say that ‘Observable \mathbf{A} has a certain value belonging to a set σ ’. Propositions about states are of the form “The system is in a state of this or that type” (in a pure state, in some mixture of pure states, in a state such that...). A *state-attribution proposition* [\mathbf{A} ,

σ] gives a probability of the value-attribution proposition, it states that \mathbf{A} will have a value in σ , with a certain probability. *Value-states* are specified by stating which observables have values and what these values are. *Dynamic-states* state how the system will develop. This is endowed with the following interpretation:

“The interpretation says that, if a system X has dynamic state φ at t , then the state-attributions $[\mathbf{A}, \sigma]$ which are true are those that $Tr(\rho \mathbf{P}_{\sigma}^{\mathbf{A}}) = 1$. [$\mathbf{P}_{\sigma}^{\mathbf{A}}$ is the projector over the corresponding subspace.] About the value-attributions, it says that they cannot be deduced from the dynamic state, but are constrained in three ways:

1. If $[\mathbf{A}, \sigma]$ is true then so is the value-attribution $\langle \mathbf{A}, \sigma \rangle$: observable \mathbf{A} has value in σ .
2. All the true value-attributions should have Born probability 1 together.
3. The set of true value-attributions is maximal with respect to the feature (2.)” (van Fraassen, 1991, p. 281)

This distinction between value-attribution propositions and state-attribution propositions allows van Fraassen to face the measurement problem from a new position. The way out proposed by von Neumann, of the contradiction between the presence of various results associated to the different terms in a superposition and the appearance of only one result, is the so called “projection postulate” which determines the non-causal state transition from the quantum state into a single term. In his spirit, an observable pertaining to a system has a value if and only if the system is in a corresponding eigenstate of the observable (the eigenstate-eigenvalue link). So, the observable, say \mathbf{A} , has a value if and only if a measurement of \mathbf{A} is certain to have a certain outcome. If the outcome of the measurement is uncertain, which is the case when the state is in a superposition of eigenstates of the observable, then the observable has no value. Van Fraassen (1991, p. 279), on the contrary, proposes to emphasize this modal character of the theory via the role of the state: “[...] the transition from the possible to the actual is not a transition of

state, but a transition *described by* the state.” And to interpret the emergence of a result in a new light:

“[...] [the emergence of a result is] as if the *Projection Postulate were correct*. For at the end of a measurement of **A** on system *X*, it is indeed true that **A** has the actual value which is the measurement outcome. But, of course, the Projection Postulate is not really correct: there has been a transition from possible to actual value, so what it entailed about values of observables is correct, but that is all. There has been no acausal state transition.” (van Fraassen, 1991, p. 288)

The MI proposed by Kochen and Dieks (K-D, for short), proposes to use the so called biorthogonal decomposition theorem (also called Schmidt theorem) in order to describe the correlations between the quantum system and the apparatus in the measurement process.³ Through this theorem one is able to distinguish, given a state $|\Psi_{\alpha\beta}\rangle$ in $H=H_\alpha\otimes H_\beta$, by tracing over the degrees of freedom of the subspace H_α or the subspace H_β , between system and apparatus (for a proof of the theorem see: Bacciagaluppi, 1996, section 2.3). As noted by Kochen (1985, p. 152): “Every interaction gives rise to a unique correlation between certain canonically defined properties of the two interacting systems. These properties form a Boolean algebra and so obey the laws of classical logic.” The biorthogonal decomposition gives in this way, a one to one relation between the apparatus and the quantum system and the following

³ Given a state $|\Psi_{\alpha\beta}\rangle$ in $H=H_\alpha\otimes H_\beta$. The Schmidt theorem assures there always exist orthonormal bases for H_α and H_β , $\{|\alpha_i\rangle\}$ and $\{|\beta_j\rangle\}$ such that $|\Psi_{\alpha\beta}\rangle$ can be written as $|\Psi_{\alpha\beta}\rangle = \sum_j c_j |\alpha_j\rangle \otimes |\beta_j\rangle$. The different values in $\{|c_j|^2\}$ represent a spectrum of the Schmidt decomposition given by $\{\lambda_j\}$. Every λ_j represents a projection in H_α and a projection in H_β defined as $P_\alpha(\lambda_j) = \sum_j c_j |\alpha_j\rangle\langle\alpha_j|$ and $P_\beta(\lambda_j) = \sum_j c_j |\beta_j\rangle\langle\beta_j|$, respectively. Furthermore, if the $\{|c_j|^2\}$ are non degenerate, there is a one-to-one correlation between the projections $P_\alpha(\lambda_j)$ and $P_\beta(\lambda_j)$ pertaining to subsystems H_α and H_β given by each value of the spectrum λ_j .

interpretation: *The system a possibly possesses one of the properties $\{|a_j\rangle\langle a_j|\}$, and the actual possessed property $|a_k\rangle\langle a_k|$ is determined by the observation that the device possesses the reading $|b_k\rangle\langle b_k|$.* It is important to notice that the seemingly ad hoc move of using a preferred basis (such as the Schmidt basis) can be given a physical motivation.⁴ There is however still an important drawback to remark. By tracing over the degrees of freedom of the system, one obtains an *improper mixture*. It is well known that improper mixtures cannot be interpreted in terms of ignorance (D’Espagnat, 1976), and thus, one comes back to the problem of interpreting modalities. Following van Fraassen’s distinction between *value states* and *dynamical states*, Dieks attempts to solve the problem of putting together the seemingly incompatible character of improper mixtures and ignorance via the distinction between *mathematical* and *physical states* (Vermaas and Dieks, 1995).

From a realistic perspective, another interpretational issue which MI need to take into account is the assignment of definite values to properties. If we try to interpret eigenvalues which pertain to different sets of observables as the actual (preexistent) values of the physical

⁴ It has been proved by Dieks (1995) that, given the following two conditions:

1. One-to-one correlation: we require a one to one correlation between the definite properties of the system and the definite properties of its environment,

2. No hidden variables: the Hilbert space formalism, with the usual representation of physical magnitudes by observables, should be completely respected.

The only basis that accomplishes these two conditions is the Schmidt basis. The first demand appears as obvious when reflecting on the preconditions which allow us to talk about measurement. The second demand can be considered as a commitment to the early interpretation of Bohr, Born, Heisenberg and Pauli; to consider the quantum description as providing all there is to know with respect to atomic events.

properties of a system, we are faced to all kind of no-go theorems that preclude this possibility. Regarding the specific scheme of the MI, Bacciagaluppi and Clifton were able to derive Kochen-Specker (Kochen and Specker, 1967) type contradictions in the K-D interpretation⁵ which showed that one cannot extend the set of definite valued properties to non-disjoint sub-systems.

In order to escape KS type contradictions the modal version of Jeffrey Bub reminds of David Bohm's interpretation and proposes to take some observable, \mathbf{R} , as *always* possessing a definite value. In this way one can avoid KS contradictions and maintain a consistent discourse about statements which pertain to the sub-lattice determined by the *preferred observable* \mathbf{R} . As van Fraassen's and Vermaas and Dieks' interpretations, Bub's proposal distinguishes between *dynamical states* and *property or value states*, in his case with the purpose of interpreting the wave function as defining a Kolmogorovian probability measure over a restricted subalgebra of the lattice $L(H)$ of projection operations (corresponding to yes-no experiments) over the state space. It is this distinction between property states and dynamical states which according to Bub provides the modal character to the interpretation:

“The idea behind a ‘modal’ interpretation of quantum mechanics is that quantum states, unlike classical states, constrain possibilities rather than actualities – which leaves open the question of whether one can introduce property states [...] that attribute values to (some) observables of the theory, or equivalently, truth values to the corresponding propositions.”
(Bub, 1997, p. 173)

In precise terms, as $L(H)$ does not admit a global family of compatible valuations, and thus not all propositions about the system are

⁵ Different no-go theorems for the atomic version of the MI were also derived by Vermaas (1999b).

determinately true or false, probabilities defined by the (pure) state cannot be interpreted epistemically (Bub, 1997, p. 119). But, if one chooses, for a given state $|e\rangle$, a “preferred observable” \mathbf{R} , these properties can be taken as determinate since the propositions associated with \mathbf{R} , i.e., with the projectors in which \mathbf{R} decomposes, generate a Boolean algebra. Bub constructs the maximal sublattices $D(|e\rangle, \mathbf{R})$ included in $L(H)$ to which truth values can be assigned via a 2-valued homomorphism and demonstrates a uniqueness theorem that allows the construction of the preferred observable.

In Bub’s proposal, a property state is a maximal specification of the properties of the system at a particular time, defined by a Boolean homomorphism from the determined sublattice to the Boolean algebra of two elements. On the other hand, a dynamical state is an atom of $L(H)$ that evolves unitarily in time following the Schrödinger equation. So, dynamical states do not coincide with property states. Given a dynamical state represented by the atom $|e\rangle$ included in $L(H)$, one constructs the sublattice $D(|e\rangle, \mathbf{R})$ with Kolmogorovian probabilities defined over alternative subsets of properties in the sublattice. They are the properties of the system, and the probabilities defined by $|e\rangle$ evolve (via the evolution of $|e\rangle$) in time. If the preferred observable is the identity operator \mathbf{I} , the atoms in $D(|e\rangle, \mathbf{I})$ may be pictured as a “fan” of its projectors generated by the “handle” $|e\rangle$ (Bub, 1992, p. 751) or an “umbrella” with state $|e\rangle$ again as the handle and the rays in $(|e\rangle)^\perp$ as the spines. When observable $\mathbf{R} \neq \mathbf{I}$, there is a set of handles $\{|e_{ri}\rangle, i = 1\dots k\}$ given by the nonzero projections of $|e\rangle$ onto the eigenspaces of \mathbf{R} and the spines represented by all the rays in the orthogonal complement of the subspace generated by the handles. When $\dim(H) > 2$, there are k 2-valued homomorphisms which map each of the handles onto 1 and the remaining atoms onto 0. The sublattice (that changes with the dynamics of the system) is a partial Boolean algebra, i.e., the union of a family of Boolean algebras pasted together in such a way that the maximum and minimum elements of each one, and eventually other elements, are

identified and, for every n -uple of pair-wise compatible elements, there exists a Boolean algebra in the family containing the n elements. The possibility of constructing a probability space with respect to which the Born probabilities generated by $|e\rangle$ can be thought as measures over subsets of property states depends on the existence of sufficiently many property states defined as 2-valued homomorphisms over $D(|e\rangle, \mathbf{R})$. This is guaranteed by a uniqueness theorem that characterizes $D(|e\rangle, \mathbf{R})$ (Bub, 1997, p. 126). Thus constructed, the structure avoids KS-type theorems. Then, given a system S and a measuring apparatus M ,

“[...] if some quantity \mathbf{R} of M is designated as always determinate, and M interacts with S via an interaction that sets up a correlation between the values of \mathbf{R} and the values of some quantity \mathbf{A} of S , then \mathbf{A} becomes determinate in the interaction. Moreover, the quantum state can be interpreted as assigning probabilities to the different possible ways in which the set of determinate quantities can have values, where *one particular set of values represents the actual but unknown values of these quantities.*” (Bub, 1992, p. 750, emphasis added)

The problem with this interpretation is that, in the case of an isolated system, there is no single element in the formalism of QM which allows us to choose an observable, \mathbf{R} , rather than other. This is why the move seems flagrantly *ad hoc*. Were we dealing with an apparatus, there would be a preferred observable, namely the pointer position, but the quantum wave function contains in itself mutually *incompatible representations* (choices of apparatuses) each of which provides non-trivial information of the state of affairs.

“[...] the change in the quantum state $|\psi\rangle$ manifests itself directly at a modal level –the level of possibility rather than actuality– through the determinate sublattice defined by $|\psi\rangle$ and position in configuration space as the preferred determinate observable.” (Bub, 1997, p. 170)

Finally, the last version we will discuss here is the atomic MI, due to Guido Bacciagaluppi and Michael Dickson (1997). It intends, via a factorization, to separate the state space of the system H in disjoint spaces H_k . A factorization Φ of a Hilbert space H into a tensor product of two Hilbert spaces $H_1 \otimes H_2$ is given by an equivalence class of isomorphisms differing only by a basis transformation of the factor spaces onto themselves. It may be proved that there are many different factorizations. The question becomes now whether, by letting Φ vary, the definite properties pertaining to the different factorizations will admit a truth valuation. Bacciagaluppi has proved that this question must be answered negatively because these properties include the set of properties for which KS have shown that it is not allowed an homomorphism to the Boolean algebra $\mathbf{2}$ (Bacciagaluppi, 1995).⁶ In order to escape this no-go theorem, Bacciagaluppi and Dickson assume that there exists in Nature a special set of disjoint sub-spaces H_k which are the building blocks of all physical

⁶ In the K-D interpretation one considers arbitrary factorizations as defining systems to which one can ascribe definite valued properties. K-D are able to ascribe properties to every quantum mechanical system, that is, to any subsystem appearing in any possible factorization of the Hilbert space of the Universe. As noted by Vermaas (1999a), the KS theorem by Bacciagaluppi (1995) is a constraint on any explicit rule correlating properties of subsystems belonging to different factorizations. In order to derive a contradiction Bacciagaluppi takes a composite ω defined on a 9-dimensional Hilbert space and considers a number of factorizations $\omega = \alpha_i \beta_i$, $i = 1, 2, \dots$ of ω in subsystems $\{\alpha_i\}$ and $\{\beta_i\}$ defined on three dimensional Hilbert spaces. Then he considers the core properties $\{P^{a_i}\}$ and $\{P^{b_i}\}$ ascribed to these subsystems by the K-D MI. Next, he ascribes these properties via Property Composition (If we have the system $\delta = \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ then the eigenprojector assigned to the molecule $\alpha_i, \alpha_j, \dots, \alpha_k$ will be $P^{j \dots k} = P^i \times P^j \otimes \dots \otimes P^k$ the products of the eigenprojections of the states of the atoms in δ . The core property ascription to δ assigns the value 1 to the projection $P^i_a \otimes P^j_b \otimes \dots \otimes P^k_c$ if and only if the core property ascription to the atoms in δ assigns simultaneously the value 1 to $P^i_a \otimes P^j_b$, etc.) to ω . Bacciagaluppi is able to prove that all the properties ascribed to ω include the set of properties for which the KS theorem shows that it does not allow an homomorphism to the Boolean algebra of $\{0,1\}$.

systems; i.e. a *preferred factorization* of the Hilbert space of the whole Universe:

“[...] we note that the idea of a preferred factorization is not, perhaps, as ad hoc as it might first appear. After all assuming that the universe is really made of, say, electrons, quarks, and so on, it makes good sense to take these objects to be ‘real’ constituents of the universe, i.e. the bearers of properties that do not supervene on the properties of subsystems.” (Bacciagaluppi and Dickson, 1997, p. 3)

It is important to notice that in this interpretation the structure of the probability assignment becomes classical, i.e. one can define a classical joint probability distribution for any set of chosen properties. As a consequence, probability can be interpreted in terms of ignorance. Bacciagaluppi’s account of MIs (Bacciagaluppi, 1996), appears in an analogous fashion to Bub’s proposal, closely related to Bohm’s causal interpretation.

“The properties possessed by a system in the modal interpretation are possessed in addition to the properties possessed by the system according to quantum mechanics. It is thus natural to call these properties ‘hidden variables’. Hidden variables theories do not represent a return to classical, pre-quantum physics. Indeed, the no-go theorems for hidden variables theories show not that hidden variables are impossible, but that they must be in important ways different from classical physics (e.g. they are non-local). On the other hand, hidden variables theories always restore a classical way of thinking about what there is. In particular, the logical and probabilistic structure of a hidden variables theory is always classical: there is no ‘complementarity’ of hidden variables, and probabilities are rigorously Kolmogorovian.” (Bacciagaluppi, 1996, p. 74)

In the atomic MI Bacciagaluppi’s theorem does not apply because one denies from the start the possibility of choosing a definite factorization for the system, i.e. to assume that one can freely factorize any given system into pairs of subsystems. However, Clifton (1996) has also proven

that, even in the case one has a definite factorization, one does not yield a proper property ascription. Clifton considers a composite system ω which can be factorized into only two subsystems α and β . By ascribing properties with the KD MI to both subsystems α and β and the complete system ω and by employing property composition, he derived that ω possesses a set of properties for which Boolean valuation is not allowed. Furthermore, Vermaas (1999) has also developed a no-go theorem for the atomic version (see also: Dieks, 1998).

3 Metaphysically Tenable Interpretations of Quantum Mechanics?

Within MI the questions related to the possibility of providing a “metaphysically tenable interpretation” have been discussed and while some versions take metaphysical presuppositions as the very starting points of departure, others present a much more agnostic position regarding metaphysical principles. Van Fraassen and Dieks positions, for example, remain close to the tradition inaugurated by Niels Bohr and his interpretation of QM. The relation of van Fraassen’s interpretation to the orthodox view can be seen as a consequence of maintaining a “conservative” position regarding the values of definite properties:

“The interpretational question facing us is exactly: in general, which value attributions are true? The response to this question can be very conservative or very liberal. Both court later puzzles. I take it that the Copenhagen interpretation – really, a roughly correlated set of attitudes expressed by members of the Copenhagen school, and not a precise interpretation – introduced great conservatism in this respect. Copenhagen scientists appeared to doubt or deny that observables even have values, unless their state forces to say so. I shall accordingly refer to the following very cautious answer as the Copenhagen variant of the MI. It is the variant I prefer.” (van Fraassen, 1991, p. 280).

In van Fraassen's empiricist account, *actuality*, the *hic et nunc*, is the only aspect which must be considered by the physicist.

“To be an empiricist is to withhold belief in anything that goes beyond the actual, observable phenomena, and to recognize no objective modality in nature. To develop an empiricist account of science is to depict it as involving a search for truth only about the empirical world, about what is actual and observable.” (van Fraassen, 1981, pp. 202-203)

Metaphysics remains outside the scope of the Copenhagen variant and only actual measurements exposed in observable phenomena need to be taken into account. Van Fraassen anti-metaphysical position relies on his constructive empiricist stance, according to which, the aim of science is to provide theories that are empirically adequate. From within this stance Van Fraassen remains agnostic regarding the interpretation of *possibility* – which he considers just a theoretical device whose only purpose is to provide a consistent account of that which is observed.⁷ Dieks also remains agnostic regarding the interpretation of possibility. However, his realistic starting point – as expressed in his first set of papers (Dieks 1988a; 1988b; 1989) – seems to provide a strong tension between the interpretation of Ψ as a mathematical element which talks about the Universe in terms of *possibilities* and the meaning of this Universe in terms of an objective account of physical reality.⁸ Dieks, attempts to discuss about “systems which possess properties” making explicit at the same time the fact that one should remain within the formal scheme of orthodox QM. “The ascribed properties are thus not fixed by something which is not part of the quantum formalism – they are not put in ‘by

⁷ This is quite an analogous move to that of Bohr who considers the quantum wave function, Ψ , as an algorithm.

⁸ This tension appears explicitly if we acknowledge the fact that the *possibility* which arises from the orthodox formal structure of QM is by no means “classical” (Domenech, Freytes, de Ronde, 2006; de Ronde 2010).

hand', for instance." (Vermaas, 1999a, p. 43) Dieks's attempt, contrary to Bub's and the atomic interpretation, relies on the formalism itself, its structure and symmetries. The versions of both van Fraassen and Dieks do not seek, necessarily, to construct a dynamical picture and are only worried to build a consistent bridge between the empirical data and the quantum-theoretical framework.

Contrary to Van Fraassen and Dieks, Rob Clifton is one of the most clear proponents of taking into account "right from the start" metaphysical considerations when discussing the possible interpretation of the quantum formalism. According to Clifton, MIs "aim to tell a systematic story about what the categorical properties of quantum systems are that is not built upon the eigenstate eigenvalue link." Clifton states that MIs stand on a series of desiderata, the first of which is that: "The set of categorical property ascriptions to systems in any given quantum mechanical situation at any given time should be metaphysically tenable." (Clifton, 1996, p. 382). Clifton makes explicit this characterization:

1. The set of categorical property ascriptions to systems in any given quantum mechanical situation at any given time should be metaphysically tenable.
2. It should be possible for the probabilities dictated by the quantum formalism for measurement results to be recovered as measures over the different possible property ascriptions applicable in the special case of measurement interactions.
3. It should be possible to give a sensible deterministic or stochastic dynamics for the evolution of properties and their probabilities over time that is consistent with the Schrödinger evolution of quantum states.
4. Property ascriptions to macroscopic objects should be sufficient to recover our everyday perceptions of those objects.

5. It should be possible to achieve all of the above without necessarily breaking Lorentz invariance.

The conditions imposed by Clifton advance then by characterizing the relation between quantum properties: “as part of satisfying desideratum (1), property intersection should be imposed on modal interpretations [...]” (Clifton, 1996, p. 382). However, if we recall the idea that MIs must remain close to the standard formalism, this seems an impossible mission to accomplish. It is far from obvious how to recover classical properties and at the same time maintain the orthodox quantum formalism. Pieter Vermaas characterizes the realist attitude in the following manner:

“[If] one adopts a [...] realist attitude towards quantum mechanics and assumes that it is a theory about electrons, protons, etc., which exist independently of us and independently of the performance of measurements, then the standard formulation can only be a beginning. In the realist conception, a true physical theory about elementary particles, aims at (literally) describing the properties of those particles as they exist out there.” (Vermaas, 1999a, p. 16)

Vermaas presents then a set of metaphysically tenable desiderata for developing MIs, but remains very cautious of imposing conditions on unobservable states of affairs.

1. The interpretation should give a description of reality in which things like positions, spin and energy are normal physical magnitudes which pertain to systems and which exist independently of the notion of observation or measurement. An interpretation should ascribe properties to systems, meaning, that it should yield a fully flagged theory of these properties.
2. The description of reality given by an interpretation should be empirically adequate, meaning, that the interpretation should

reproduce the predictions of the standard formulation of QM with regard to the outcomes of measurements.

3. The interpretation should give a metaphysically tenable description of the magnitudes and properties of systems.

According to Vermaas:

“The third demand that a modal interpretation should yield a metaphysically tenable description of reality surpasses the first two demands because a fully developed and empirically adequate description of reality can still give a totally weird and unacceptable description of the properties of non-observed quantum systems. [...] [B]ecause modal interpretations describe states of affairs which are in principle unobservable, one should be careful about discarding modal descriptions of reality as metaphysically untenable. And it seems to me that it is incorrect to impose intuitions about descriptions of what is observable on descriptions of what is, in principle, unobservable. The criteria I propose for metaphysical tenability are thus very sparse:

Consistency

The description of reality should be free of contradiction.

Internal Completeness

The description of reality by an interpretation should be complete with regard to the standard set by that interpretation: that is, an interpretation should deliver the description that it promises to deliver.” (Vermaas, 1999a, p. 34)

Regarding the first point proposed by Vermaas, it is interesting to note that *demand 1*, might be regarded as metaphysical demand. Vermaas states that MIs should “give a description of reality in which things like positions, spin and energy are normal physical magnitudes which pertain to systems and which exist independently of the notion of observation or measurement.” The question is, why should we have necessarily such a conceptual structure surrounding QM? It might be the case that QM could

be interpreted in terms of properties and systems, and that the properties behave as in the case of classical physics, but, isn't classical physics already a (metaphysical) construction? A structure which is not self evident. So what is the justification for asking the theory to accomplish demands? Isn't this already a metaphysical stance?

4 Discussion: Formal vs. Metaphysical Constraints

We believe that the most important distinction which one can draw in the huge interpretational map of the quantum – from which, for obvious reasons, instrumentalist positions are left aside – deals with the position one takes with respect to metaphysics. One might characterize the map of the quantum in the following manner. On the one hand we have a first group which attempts to start with a quite clear metaphysical picture related most of the time to classical notions such as space-time, causality, objects, etc. Depending on which of these notions is taken as most important, other ones need to be dropped and even the formalism might be subject of development and transformation. Starting from metaphysical presuppositions in the interpretation of QM, one seems to be immediately forced to change the formal structure in order to reproduce the desired features of this “new theory”. Such is the case of Bohmian mechanics which, in order to discuss about ‘positions’ and ‘fields,’ is forced to change the formalism with *ad hoc* moves, moves which can be only justified in relation to the *prior* metaphysical commitments. On the other hand, a second group also interested in the metaphysical question regarding QM attempts to begin “right from the start” with the successful mathematical formalism in its orthodox form, trying to learn about its structure and characteristics in order to find a metaphysical scheme which is able to fit the formalism. According to this position we need to understand what is the quantum formalism telling us about the world, and we should not believe that we already know what the world is like. We might consider the first group as going from

metaphysics into the formal structure while the second group goes from the formal structure into the metaphysical scheme.⁹ Also within MIs we find an equivalent set of versions, which can be divided according to the previous distinction.

The first group attempts to start from the orthodox mathematical formalism and find a consistent interpretation through the development of a metaphysical scheme, we shall call these versions *Modal Interpretations (which start from) the Mathematical Formalism* (MIMF). The first example we can take of MIMF is van Fraassen's Copenhagen variant. Another example of MIMF is Dieks proposal, whom although started with a realistic interpretation of systems with properties, is not unwilling to change his metaphysical presumptions into less orthodox ones, as in the case of his relational version of the MI (Bene and Dieks 2002; Dieks 2009).

“The proposal is to conceive the mathematical structure of quantum theory as a representation of the physical structure of the world [...] there is an additional source of meaning from the relation with experience. A number of substructures of the total structure are identified with isomorphous structures constructed from empirical data. As a result the structure makes contact with the world [...]” (Dieks, 1989, p. 1416)

Following van Fraassen's consideration, that outside of measurement contexts “anything is possible” (van Frassen 1991, p. 294), in these interpretations there is no dynamical evolution from the *dynamic state* to the *value state*. Van Fraassen and Dieks remain very cautious regarding the conditions applied to the properties discussed in the quantum

⁹ Strange as it might seem the most “metaphysical” positions regarding this map come from approaches which have an important reputation in Anglo-Saxon regions, Many Worlds interpretation in England and Bohmian mechanics in the States. While the less metaphysical positions which relate to quantum logic and MIs have their center of action in the Continent.

formalism. The mode of existence regarding the properties in the dynamical state (or in the mathematical state) is what provides a formal picture which violates property composition and property intersection, – the metaphysically tenable conditions which properties must respect according to Clifton – and which cannot be interpreted in terms of preexistent actuality. Both van Fraassen and Dieks remain within the orthodox position that the state of the system is all there is to know and there are no hidden variables to be added.

In both Dieks and van Fraassen’s account we must stress the “openness” to describe physical reality in terms of what the formalism is “pointing to” and not presuppose too much about reality itself. Both are ready to leave aside the classical metaphysical presuppositions and advance towards a different story of what QM is telling us about the world. According to Dieks:

“[...] there is no visualizable model encompassing the whole structure [of quantum theory], the demand that there should be a visualizable model would be tantamount to demand that classical physics should determine the conceptual tools of new theories. This would deny the possibility of really new fundamental theories, conceptually independent of classical physics” Dieks (1989, p. 1417)

Such original paths can be witnessed in the relational modal version of Bene and Dieks (2002) (see for discussion: de Ronde 2003; Dieks 2009), and also in van Fraassen’s analysis of Rovelli’s relational interpretation (van Fraassen, 2009).

As in the general set of interpretations, within the so called modal scheme there is, as we have seen above, a sub-set of versions which begin their analysis with much more solid metaphysical conditions than Dieks and Van Fraassen’s proposals. We shall call these versions, *Modal Interpretations (which start from) Metaphysical Principles* (MIMP). The main aim of hidden variable type theories is to provide a (classical) metaphysically tenable characterization of what is going on according to

QM. Although these conditions might vary from one version to the other, the will to retain a picture in terms of classical physical theories is always present. In this sense, Bub's Bohmian version and the atomic proposal of Bacciagaluppi and Dickson can be accounted for what we call here MIMP.

In the case of MIMP there are – as in the case of Bohm – explicit formal deviations from the orthodox formulation of QM. Bacciagaluppi and Dickson regard MIs as referring to actual properties and this is why they look for a dynamical picture that governs the evolution of these properties. Within the atomic version they attempt to find a dynamical structure which can provide for both, the empirical adequacy of the orthodox formulation, and an explanation of the path from the *possible* to the *actual* by developing a stochastic scheme (Bacciagaluppi and Dickson, 1997). In a certain sense, as also noted by Ruetsche, these attempts institute not only an interpretation of the formal structure of QM, but a new theory in itself. This important difference in the account provided by the versions of MIs draws a clear distinction which was characterized by Laura Ruetsche (2003) in terms of *Modal Interpretations with Semantic Probabilities* (MISP) and *Modal Interpretations with Hidden Variables* (MIHV). Ruetsche takes here a stance and argues for MIHV by stating that: “I urge that we adopt a principle of leeway according to which the interpretation of QM needn't be a purely semantic project. This principle frees interpretations from the obligation to adjust their semantics to the state space of QM innocently construed; they may fiddle with that state space, or unitary dynamics, or both.” Contrary to this position, there are many who claim that changing the formalism is not part of “interpreting” a theory. As noticed by Healey: “[...] I cannot accept a hidden-variable theory as an interpretation of quantum mechanics. A hidden-variable theory is, fundamentally, a separate and distinct theory from quantum mechanics. To offer such a theory is not to present an interpretation of quantum mechanics, but to change the subject.” (Healey, 1989, p. 24)

Our own stance attempts to remain within the metaphysical question which relates physics with Being, taking seriously at the same time the critics of 20th century analytic philosophy. QM is a very good region of thought in which our methodology can be expressed. According to our stance we must remain close to the orthodox formalism simply because it seems there is no good reason – apart from metaphysical presumptions – to change it. Empirical success is the most important element in order to accept a physical theory, and the quantum formalism is above all, already extremely successful in empirical terms. Reality should not be a pre-established concept nor a prejudice in observing and relating empirical data, but rather a goal concept which should be transformed and developed. We should not expect reality to be as we would like it to be. We must constantly revise the conceptual framework with which such a description is expressed. The physicist and the philosopher should remain in humble position, not presupposing that they already know what reality is about.

As noticed by van Fraassen “a philosophical position can consist in something other than a belief in what the world is like. [...] A philosophical position can consist in a stance (attitude, commitment, approach, a cluster of such – possibly including some propositional attitudes such as beliefs as well). Such a stance can of course be expressed, and may involve or presuppose some beliefs as well, but cannot be simply equated with having beliefs or making assertions about what there is.” We are now ready to advance in a series of points which characterize our own stance regarding the possible interpretation of quantum theory:

1. QM makes reference/expresses a feature of reality. We remain open to new possibly revolutionary features of reality. This means we are not ready to accept the classical concepts as determining the conceptual structure of quantum theory. As noted by Dieks (1989, p. 1417): “This would deny the

possibility of really new fundamental theories, conceptually independent of classical physics.”

2. The formalism of QM is able to provide (outstanding) empirically adequate results. Empirical adequacy determines the success of a theory and not its commitment to a certain presupposed conception of the world. Thus, it seems to us that the problem is not to find a new formalism. On the contrary, as also remarked by Dieks in relation to MIs, the ‘road signs’ point in the direction that we must stay close to the orthodox quantum formalism.
3. Until today, there seems to be no coherent interpretation of QM. This deals with the impossibility to relate the mathematical structure of the theory to a conceptual structure which allows to account in a coherent manner to quantum experience.
4. In order to learn about the limits of the classical concepts – such as: properties, apparatuses, systems – from QM, we need to work out the limits of the formalism, the symmetries, its invariances. To learn about what the formalism is telling us about reality and how to express this in more adequate terms, we might be in need of creating new concepts.

Within our stance, MIs can be understood as positing the limits of classical language with respect to the quantum formalism. Within our methodological scheme, MIs understood in this way are extremely important because, in order to go beyond, we first need to learn about the limits which we are dealing with. The problem regarding the interpretation of QM relates to the finding of an adequate metaphysical scheme, or in other words, we still need to answer the question: what is QM talking about?

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