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GETTING OUT OF A HOLE: IDENTITY, INDIVIDUALITY AND STRUCTURALISM IN SPACE-TIME PHYSICS

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1. Introduction

For the past fifteen years or so, there has been considerable philosophical discussion over the question whether quantum particles can be regarded as individuals or not (see French 1989, 1998, 2000; van Fraassen 1989; Huggett 1997). Typically the discussion proceeds from a consideration of the role played in quantum statistics by particle permutations. We begin by taking particles of the same 'kind', which are understood to be indistinguishable, in the sense of possessing the same 'intrinsic' or state-dependent properties. We then consider the distribution of these particles over states - two particles over two one-particle states, say - and it is assumed that each resulting arrangement is accorded equal probability. In classical statistics the situation where we have one particle in each state is given a weight of two, corresponding to the two arrangements or complexions that may be formed by a permutation of the particles. This gives us the standard Maxwell-Boltzmann statistics, of course. That a permutation of the (indistinguishable) particles is included in the count of possible arrangements is taken to imply that the particles are individuals, in some sense. How one understands this individuality can then be spelled out in familiar terms: either that of some underlying 'haecceity' - also known as 'primitive thisness' and articulated in terms of self-identity - or, more typically, via some form of the Identity of Indiscernibles, with the spatio-temporal location of the particles as the privileged distinguishing property (see French 1989, 1998 for further details).

Now we can see how the examination is going to proceed in the quantum case. Here, standardly, we have two forms of statistics of course, but in both cases the above situation of one particle in each state is given a weight of one. This is standardly taken to reflect the fact that arrangements obtained by particle permutations do not feature in the relevant counting in quantum statistics. The implication, then, is that the particles can no longer be considered to be individuals, that they are, in some sense, 'non-individuals'. Historically this implication emerged in tandem with the formal development of quantum theory itself and for many years it remained the 'received view' of the issue (Hesse 1963; Post 1963).

Metaphysically this 'non-individuality' has been expressed as a loss of self-identity and thus has been perceived as a challenge to standard set theory[1]. Responding to this challenge, Dalla Chiara, Toraldo di Francia and Krause have developed non-classical set theories which, they claim, more adequately represent quantum objects than the standard versions (Dalla Chiara and Toraldo di Francia 1995; Krause 1992). Krause's 'quasi-set theory', in particular, offers a semantics for da Costa's formalization of the idea that quantum particles have no identity through his 'Schrödinger logics', in which the expression $x = y$ is not a well-formed formula in general. The philosophical issues of how we can still refer to such objects and, more fundamentally, perhaps, how they can even be understood as *objects* in the first place have also been explored.

This is only one half of the metaphysical story, however and it turns out that an alternative to the 'received view' can be constructed. The implication from quantum statistics to non-individuality can be resisted by giving another account of the reduction in statistical weight attaching to the distribution of one particle in each state in our example above. Basically, another explanation for this reduction can be given in terms of the inaccessibility of certain states: if, in this case, the restriction is imposed that the state of the system be either symmetric or anti-symmetric then only one of the two possible states formed by a permutation of the particles is ever available to the system and so the statistical weight corresponding to the distribution of one particle in each (one-particle) state is half the classical value. On this view, the reduction in weight has nothing to do with the supposed non-individuality of the

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[1] One might recall at this point Cantor's understanding of a set in terms of '... collections into a whole of *definite*, *distinct* objects of our intuition or of our thought.' (Cantor, 1955, p. 85, my emphasis).

particles and so one can continue to regard them as *individuals* which are simply prevented from occupying certain states (French 1989; 1998). Of course, how one understands this individuality *metaphysically* may still be problematic. Given that the particles are still regarded as possessing the same set of intrinsic properties, they cannot be individuated in terms of these and hence it has been argued that the Principle of Identity of Indiscernibles (PII) is violated in quantum mechanics (French and Redhead 1988)[2]. Articulating this individuality in terms of spatio-temporal location runs into the well known problems to do with trajectories in quantum mechanics. This leaves 'haecceity' or primitive thisness and one is left in the rather uncomfortable position of having to insist that although there is no possible way of *distinguishing* the particles, nevertheless they can still be regarded - metaphysically - as individuals.

Rather than continue to pursue this particular theme, I want to consider a different issue: can we transfer the terms of this discussion to space-time points? As I shall try to indicate, the answer to this question has an important bearing on discussions of the ontological status of space-time.

2. Distinguishability, Individuality and Space-Time Points

First of all, there is an obvious difficulty in addressing the above question. In the case of physical objects, whatever account of individuality we adopt, we begin with distinguishability and then either make the move from epistemology to ontology, or equate the two. In the case of space-time points, there is an issue as to whether they can be regarded as *distinguishable to begin with.*

In both classical physics and special relativity, where the space-times - Euclidean and Minkowski respectively - are flat, there is nothing to distinguish one space-time point from another. As Anderson has famously put it:

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[2] Unless one were to accept some form of 'empirically superfluous' property which serves to distinguish and thus individuate the particles (van Fraassen 1991).

> The distinguishing feature of a particular point of ... space-time is that it has no distinguishing features; all points of space-time are assumed to be equivalent.' (Anderson 1967, p. 4)

In the context of General Relativity, on the other hand, there may be the possibility of the curvature of space-time proving capable of distinguishing the points. This suggestion has been raised in the context of the debate over whether curvature is intrinsic to space-time or not, which in turn can be situated in the broader debate concerning what we take 'space-time' itself to be.

In a fascinating and rich dialectic between Grünbaum and Stein, one of the more central of the issues covered concerns the ontological status of the space-time metric. In particular, the question addressed is 'whence does the space-time manifold get its particular metric structure?' (Grünbaum 1977, p. 332). Typically, but not necessarily, substantivalists will argue that this structure is intrinsic or, in some sense, 'internal', to space-time, whereas relationists will respond that it is imposed from without, by appropriate metric standards suitably embodied. Stein, however, raises the issue as to whether we can even appropriately *distinguish* this metric structure, as somehow separate from space-time, to begin with; that is, he is concerned that the question regarding the ontological status of the metric may beg too many questions both with regard to its relationship with the space-time, however construed, and with regard to our (theoretical) access to space-time (*ibid.*, pp. 327-328). There is a contrast here, Stein claims, between everyday objects, such as a globe sitting on a desk, and space or space-time. In the former case, we can sensibly ask whether the globe possesses certain *intrinsic* metrical properties[3] because we have access to it, independently of these properties. The problem is, he insists, we have no such independent access to space-time. Thus Stein writes,

> ... if we ask (assuming Newtonian physics) whether "equality of time-intervals" is a relation *intrinsic to the space-time manifold*, and if this is construed (roughly) to mean "whether that relation is involved in the structure of *the space-time manifold itself, considered apart from all*

[3] Such as that of having a surface which is a Riemannian 2-manifold of constant positive curvature (Grünbaum 1977, p. 328).,

> *other entities*," the question at once arises of how to explicate the notion of "the space-time manifold itself," and of the *conceptual* line between it and "all other entities." I see no way to confront the former question independently of the latter; and yet the converse may also seem to hold: that we cannot give a conceptual explication of "the space-time manifold" without begging the question of its intrinsic properties. (quoted in Grünbaum *ibid.*, p. 329)

In response, Grünbaum appeals to the example of a man named 'Jack' who happens to be an uncle, in order to draw the well known distinction between intrinsic and extrinsic properties (*ibid.*, pp. 332-333). Thus, Jack possesses the property of being an uncle only in virtue of there being someone else, 'external' to him, of whom he is their uncle. 'Unclehood' is ontologically dependent on a relation to an object external to Jack, whereas the property of bipedality is not. In other words, unclehood is extrinsic, whereas bipedality is intrinsic. Now, continues Grünbaum,

> ... even after Jack has been explicitly characterized as an uncle upon having been *identified* as "Uncle Jack," such identification does *not at all beg the question* whether his unclehood is ontologically intrinsic to him in the manner of his genuinely monadic property of bipedality! (*ibid.*, p. 333)

If we are careful to make the distinction between distinguishability and individuality, the use of certain properties in *distinguishing* an object should not then beg any questions as to the ontological status of such properties.

This becomes even clearer in the further example Grünbaum gives of distinguishing two individuals, in his case, Presidents Ford and Giscard d'Estaing (remember them?!). Each can be characterized and identified in terms of certain properties - President of the U.S.A. and President of France respectively - and, Grünbaum insists, regardless of which particular properties effect these identifications, one can determine in a non-question begging manner which properties are intrinsic and which are not. What is important for distinguishing the two is not whether the properties involved are intrinsic or extrinsic, but that there is *some* difference between their properties:

> *By invoking a particular set of properties initially in a definite*

> *description to identify these (human) objects, one does* not *thereby prejudge or determines the degree (monadicity vs. polyadicity) which these and the properties of theirs will turn out to have subsequent to the identification.* (*ibid.*, p. 335; emphasis in original)

This analysis is then carried over to the case of space-time and Grünbaum insists that the fact that certain metrical properties feature in the identifying characterization of space-time does not prejudge the question whether such properties are intrinsic, by analogy to Jack's bipedality, or extrinsic and ontologically relational, by analogy with his unclehood. Granted that we may have *easier* epistemic access to globes on desks, say, than to the metrical feature of space-time, this difference

> .... cannot serve to sustain Stein's charge that owing to the unavailability of an independent ostensive definition, any theoretical identification of physical space-time ineluctably begs the question concerning the ontological intrinsicality (monadicity, absoluteness) of its identifying properties.' (*ibid.*, p. 337)

We shall come back to Stein's response below, since both this response and the above debate in general offer illuminating comparisons with the situation regarding particles. For the moment, let us return to the question of the individuality of space-time points. In the case of particles our metaphysical considerations were grounded in a feature of scientific practice, namely the counting as distinct of arrangements formed by permuting particles over states. Is there anything analogous in the physics of space-time? Grünbaum has argued that there is and that it can be found in the physicists' generation of so-called 'covering spaces' by the 'disidentification' or explosion of points in a given manifold (Hawking and Ellis 1973, p. 181).

Consider again an example of an 'ordinary' object: a hemisphere sitting on a table in Euclidean space (*ibid.*, p. 365). This can be turned into a model of elliptic 2-space by identifying the antipodal equatorial points of the surface. Nevertheless, in this case, Grünbaum insists that it is both *meaningful* to assert that the antipodal points are in fact distinct, since they coincide with different points of the table, for example, and that, furthermore, we are *epistemologically* able to determine that they are distinct. The suggestion, then, is that both of these analyses carry over to the physics of space-time.

Thus, as Grünbaum notes, Schrödinger considered the warrant for making a similar identification of the antipodal points of a pseudo-sphere in De Sitter space-time to give an elliptic interpretation. Moving the other way, one can 'disidentify' the points of a manifold to give a *topologically different* covering manifold. Hawking and Ellis appeal to just such a procedure in order to obtain the property of time orientability in the case in which a particular space-time lacks it and take it as an assumption that either the space-time is time-orientable or one can deal with the time-orientable covering space (*op. cit.*; Grünbaum *op. cit.*, p. 366). How should we regard these different models? Glymour argues that they must be understood as contradictory and irreconcilable on the grounds that the alternative topologies depend on the basic individuals of the models and that such differences are matters of truth or falsity (Glymour 1977). Hence it is meaningful to ask whether the space-time points of 'our' universe are distinct, in the above sense, or not and consequently, meaningful to ask which of the topological alternatives we find ourselves in. Having established this much, it is then a further epistemological question whether we can ever *discover* which space-time we are in (Grünbaum *op. cit.*, pp. 365-366). Both Glymour and Malament have argued that for certain space-time models, there can be no empirical evidence which could ever resolve this issue[4].

Finally, Grünbaum asks: 'what criteria of identity or distinctness for [space-time points], if any, can give *physical meaning* to the required formal disidentifications at the ontological level of postulated space-time theory?' (*op. cit.*, p. 366) In particular, an adequate defense of the claim that the above questions are at least meaningful '... depends on the provision of a viable criterion of individuation for [space-time points] which are prima facie so much alike with respect to their monadic properties.' (*ibid.*)

So, what criteria of individuation might there be? Let us consider the

[4] This raises obvious problems for the realist (Torretti, personal communication). The anti-realist could either take the positivist route and simply deny that it is meaningful to ask which model we are 'in', or, more sophisticatedly perhaps, take the path of the constructive empiricist and agree that it is indeed meaningful to ask such a question, since there is a truth of the matter, but deny that we could ever know that truth. The realist might respond by appealing to some form of structuralism as indicated towards the end of this paper.

Principle of Identity of Indiscernibles first of all. As Stein has noted, Newton also accepted this principle and took it to apply to space and time (Newtonian Space-Time, p. 184; Grünbaum *op. cit.*, p. 364). On this basis, in order to be considered as individuals, the points of space, time and space-time must possess some distinguishing property or relation. For Newton, the 'parts' of space could be regarded as distinguished and, hence, individuated by their 'internal' relations:

> ... just as the parts of duration derive their individuality from their order, so that (for example) if yesterday could change places with today and become the latter of the two, it would lose its individuality and would no longer be yesterday, but today; so the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other. The parts of duration and space are understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered.' (Newton in Hall and Hall *op. cit.*, p. 136; cf. Principia, Cajori ed. p. 8)

But, of course, it is the apparent unavailability of any independent notion of the 'position' of a point of space that led Leibniz to deploy the Identity of Indiscernibles *against* the Newtonian absolutist view, arguing that since on this view such points are *indistinguishable* in terms of their intrinsic, monadic properties, they must all be *identical*, in the strict sense. As this is absurd, the Leibnizian will insist that the notion of space as composed of such points must be rejected in favour of a relationist view in which 'external' relations involving nonspatial bodies serve to individuate the points of our model, regarded as a mathematico-physical description only.

What about the situation in General Relativity? Can the curvature of relativistic space-time provide sufficient heterogeneity to allow for the points to be distinguished, and hence individuated via PII? Grünbaum considers a method of constructing an intrinsic coordinate system using the metric which, if it could be applied globally, might be up to the job (*op. cit.*, pp. 366-367). Unfortunately, he acknowledges, it can't and it isn't, since this method works only locally and even then it may not be able to individuate. Here, then, we face the problem noted above: we simply don't have a grip on the distinguishability of the space-time points in the first place and therefore don't have the basis for appealing to PII in order to guarantee their individuality.

Perhaps, then, we should abandon the search for some distinguishing features as hopeless[5] and directly assert that the space-time points are individuals, understood in terms of primitive thisness, for example. Grünbaum himself rejects such a proposal, on the time honoured grounds that the property of self-identity satisfies PII only trivially and cannot therefore be accepted as an individuating property (*op. cit.*, p. 364). However, primitive thisness can be effectively disconnected from PII and invoked as a principle of individuality in its own right, as it were. It is interesting, therefore, that Hoefer has recently identified just this notion of primitive thisness as laying at the core of a further and much discussed issue in space-time physics which, similarly to the covering space example above, also involves the generation of alternative space-time models which cannot be distinguished empirically (Hoefer 1996). This is the issue concerning the implications of the infamous 'hole argument'[6].

3. Abandoning Identity: A Response to the Hole Argument

Given a space-time model in General Relativity, another model can be generated which is identical to the first for all points outside a certain region - the hole - but not inside, through the application of an appropriate diffeomorphism to the points of the underlying manifold. If the situation regarding the hole is chosen appropriately, it turns out that the two models can agree up to a certain time but then diverge afterwards, giving what is regarded as a failure of determinism. Now, if substantivalism is understood to involve the identification of space-time

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[5] It may not be entirely. Grünbaum notes that Glymour has suggested that for certain space-times in which the two lobes of the light cone are disjoint, metric relations may serve to distinguish and hence individuate (Glymour 1972; Grünbaum *op. cit.*, p. 367). However, it is unclear whether this criterion of individuality can serve to distinguish between a space-time and its covering model and thus answer Grünbaum's question as posed above.

[6] The argument can be traced back to Einstein but in its current incarnation was spelled out in a well known paper by J. Earman and J. Norton (Earman & Norton 1987).

with the underlying manifold - on the grounds that in General Relativity geometric structures, such as the metric tensor, are physical fields 'in' space-time (Earman and Norton 1987, p. 518) - then the substantivalist must accept these two qualitatively identical models as physically distinct. But then substantivalism would seem to imply a failure of determinism, an implication which has been taken as a form of reductio of substantivalism on the grounds that if determinism fails it should do so for a reason to do with physics, not metaphysics (Earman and Norton *ibid.*, Earman 1989, p. 180; Butterfield 1989 ; Maudlin 1988) I shall be less concerned with this conclusion than with the implications of the argument as a whole for the discussion of the individuality of space-time points.

It is worth noting, as Hoefer does (*op. cit.*, pp. 7 and 8), that a diffeomorphism can be seen as a *permutation* of the points of the manifold which satisfies certain restrictions. This provides an obvious point of comparison with the situation in particle physics (for further discussion of this comparison, see Stachel forthcoming). In the hole argument a permutation of the space-time points leads to a new model, analogous to the new arrangement generated by a permutation in classical statistical mechanics. In the latter case, this new arrangement is counted as distinct and if it were not, the statistics would be very different. In the case of space-time physics, how are models generated by such diffeomorphism regarded? It turns out that physicists do not regard such models as physically distinct, since they are qualitatively indistinguishable, the only difference concerning what fields are located at what space-time points. This attitude has been interpreted as an endorsement of 'Leibniz Equivalence', which holds that two such models are equivalent in the sense of representing the same physical situation (Earman and Norton *op. cit.*). Substantivalism, of course, is committed to the denial of Leibniz Equivalence, just as the Newtonian absolutist was earlier committed to the denial of the claim that a description of the universe as it is, is equivalent to a description of the universe with its material context moved 10 feet in a given direction in space.

If such a space-time model is regarded as describing a possible world, then denying Leibniz Equivalence in the context of the hole argument implies the acceptance of haecceitism (Hoeffer, *op. cit.*, p. 15). In the present context, this doctrine holds that two space-time models may not differ qualitatively in any way, yet still differ in what they represent *de re* concerning the individual space-time points. Hoeffer goes further in claiming that since acceptance of haecceitism entails acceptance of 'primitive identity', substantivalists, in denying Leibniz Equivalence, ascribe primitive identity to space-time points:

> ... two such models can only represent different physically possible worlds if we believe that space-time points (or regions) not only exist, but have primitive identity, and so could have all of their properties systematically exchanged with the properties of other actual points ... (*ibid*.)[7]

This entailment is problematic. According to Lewis, a belief in haecceities is neither necessary nor sufficient for haecceitism in the above sense. If haecceities are understood as Adams understands them, namely as primitive thisnesses involving primitive identity, and Hoefer explicitly draws his notion of the latter from Adams, then one might assert haecceitism but deny primitive identity, on the grounds that Lewis has laid out. In that case, of course, our substantivalist is going to have to appeal to some other Principle of Individuality consistent with such grounds; one such might be 'bare' substance. This gives a rather curious account of the individuality of space-time points but it is one that the substantivalist could cling to if necessary.

Alternatively, one could try to resurrect the idea that this individuality is grounded on the properties - that is, the *structural* properties - of the space-time points themselves. Maudlin has offered a version of this idea which is a form of metric essentialism: space-time points bear their metrical relations essentially (Maudlin 1989 p. 86)[8]. This undermines the basis of the hole argument since the diffeomorphism is not now taken to generate a possible situation in the first place.

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[7] Here Hoefer appeals to the example of two dice, concerning which the question is asked: does it make sense to ask whether dice A could have all the properties of dice B and vice versa? (He may have obtained this example from Kripke, who uses it in his discussion of rigid designation). As he notes, anyone who holds that the dice are individuals, and in particular if such individuality is understood in terms of primitive identity, will answer 'yes' to this question.

[8] Maudlin claims ancestry for this view in the passage by Newton quoted above; see also Earman 1989 pp. 218-219 for further discussion.

Concomitant with this idea is a shift in what we take space-time to be: for Maudlin it is not simply the manifold but the latter *plus* the metric structure. Unfortunately, however, Norton has argued that if space-time is understood as the manifold plus further structure and if this further structure allows of certain common symmetries, then a version of the hole argument can be revived (Norton 1989)[9]. In particular, such symmetries arise in Newtonian space-time theory and in Special Relativity and General Relativity as applied to spatially homogeneous and isotropic cosmologies. As Norton says, the crucial point here is that

> ... the presence of these symmetries represents a failure of the further structure to individuate fully the points of the manifold (*ibid*, p. 60)

There are other responses to the hole argument, of course (see Butterfield 1989; further discussion can be found in Earman *op. cit.*, Chapter 9) but Hoefer's response is radical: he denies that the space-time points have primitive identity (*ibid.*, p. 15)[10]. Of course, given what we've just said, this doesn't in fact resolve the issue, since one might adopt an alternative account of the individuality of space-time points; what is required, it seems, is the denial of *haecceitism* in general. We shall return to this point shortly.

As Hoefer recognizes, this denial of primitive identity faces a number of challenges, not least of which is one bound up with our remark that an account of individuality of space-time points without primitive identity would be curious. And its curious nature is in fact brought out by the comparison Hoefer makes with the case of particles in an effort to respond to the challenge that primitive identity is surely part of what it means for something to be a substance (*ibid.*, pp. 15-18). As far as Hoefer is concerned, we do not need primitive identity to make sense of questions such as 'which (classical) particle entered which state?' since we have recourse to the space-time trajectories of the particles. He then generalizes to the example of unbiased dice and insists that the basis

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[9] Norton's argument crucially hinges on the claim that, as in the case of pure 'manifold substantivalism', incorporating further structure does not prevent the substantivalist from being committed to the distinctness of observationally indistinguishable states of affairs (*op. cit.*, p. 60).

[10] As he acknowledges, such a suggestion has been previously made by Maidens (1993).

for the assignment of the appropriate statistics is not an assumption that the dice have primitive identity, but rather the 'fact' that they have distinct continuous trajectories. Space-time points, of course, do not have such trajectories. If they did, he claims, then they might be capable of exhibiting Maxwell-Boltzmann statistics (*ibid.*, p. 17) but this would not mean that they have primitive identity. He concludes:

> What goes for points goes for particles. Even if atoms had distinct and continuous trajectories, we would not have to ascribe primitive identity to them in order to think of them as real substances. The ascription of primitive identity allows us to pose certain strange philosophical questions - but not to do any more productive work.' (*ibid.*, pp. 17-18)

As I have indicated above, arguments have been given for a similar 'loss' of individuality in particles (it should be noted that such arguments and their further discussion precede Hoefer's, of course[11]). In the absence of a similar sort of argument here, Hoefer's response seems somewhat ad hoc. He is right, of course, that we can always ground Maxwell-Boltzmann statistics in some form of individuality via spatio-temporal trajectories but adherents of haecceity will insist that the 'fact' that such entities possess trajectories pertains to their distinguishability only; their individuality rests on something else. Hoefer's failure to distinguish (!) between distinguishability and individuality is apparent from the last sentence in the above quote; of course primitive identity does no 'productive work' when it comes to distinguishability, but, its defenders would insist, it does *all* the metaphysical work in grounding individuality.

This last point is important because Hoefer adheres to the view that individuality needs no such grounding, that it can be taken as primitive. This emerges in his response to the further challenge: can we make sense of a notion of substance sans primitive identity? Not surprisingly, Hoefer thinks we can. This is problematic but let us grant that we can make sense of this notion of primitive individuality without primitive identity, how, then, are we to interpret the manifold of our space-time model? Mathematically, this manifold is a set of points possessing a certain topological structure. If we accept that the theory of General Relativity

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[11] Indeed, such arguments on the particle side motivated Maiden's rejection of individuality on the space-time side (*op. cit.*).

quantifies over these points, then it would seem that we are committed to their existence as distinct individuals (Hoefer *op. cit.*, p. 23). There are two options one could choose between at this point. One could agree that the theory quantifies over the points, but insist that they do not possess primitive identity. Since the latter involves, or rather, is nothing but, self-identity, this position requires not only a new understanding of quantification but also a non-classical logic and set theory - that is, something like quasi-set theory as applied to space-time points.

4. Stein's Structuralism

The alternative is to adopt a more fine grained approach to what the theory quantifies over and acknowledge that one could still give a realist construal of the theory in which it is interpreted as supporting the existence of space-time, but that the latter should not be identified with the manifold (we recall that in the context of the hole argument this is precisely the identification made by substantivalists)[12]. It is time to return to the Grünbaum-Stein debate, in which the issue as to what aspect of our model we take to be 'space-time' features so prominently. We recall Stein's concern that the question as to whether the metric is intrinsic or not might be a pseudo-question, since we have epistemic access to space-time only via this metric. As we saw, Grünbaum responded to this concern with his 'Uncle Jack' and two Presidents examples to indicate that we could have access to, and distinguish objects, via certain sets of properties, whilst maintaining that certain of these were intrinsic to the object. In his counter-response, Stein clarifies his concern and reveals a fundamental difference between himself and Grünbaum with regard to the ontology of space-time.

Thus with regard to the Uncle Jack and globe examples he notes that we are only able to get the process of conceptualization in terms of

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[12] It is not entirely clear which of these two alternatives Hoefer prefers. At one point he suggests that a substantivalist could deny primitive identity but not the existence of space-time points. However, he gives no indication as to how this could be formally accommodated. Moreover, he explicitly rejects the quantification over these points and the final sections of his paper seem to indicate a preference for the alternative, although the terms in which he sets it out are unclear.

intrinsic and extrinsic properties off the ground in the first place because we are already familiar with these examples, qua *objects*. They belong to a kind '... about which I possess an abundance of lore (or prejudice), namely *bodies* and *people* (antirespectively); and are quite unlike space-time, about which I happen to be deficient of such prejudice.' (*op. cit.*, p. 394)[13] As we have already said, we can distinguish, or pick out, Uncle Jack or a globe on the basis of their properties and *then* go on to discuss which of these properties are intrinsic or not but it is not clear whether we can even complete the first stage and unequivocally delineate or distinguish *space-time* to begin with. As Stein points out different delineations of space-time will give different answers to the question whether the metric is intrinsic. Thus, for example, someone might distinguish space-time as 'the mere four-dimensional differentiable manifold, independently of any further structure' (recall Hoefer's characterization of the substantivalist view above), whereas someone else might distinguish it as 'the smooth 4-manifold with the distinctions of directions at each point into *spacelike* and *causal* (timelike or null) directions.' For both such persons, the metric is extrinsic but for the second the conformal structure is intrinsic, and Stein insists that not only does he not possess any criterion for deciding between such characterizations, he sees no point in seeking one (*ibid.*, p. 395).

This is the nub of the matter. Stein understands Grünbaum as thinking of the world in terms of things or 'primary substances', whereas he thinks in terms of *structures* or *aspects of structure*. Space, or more generally, space-time, is '... *an aspect of the structure of the world.*' (*ibid.*, p. 397; his emphasis)[14]. On this view, with space-time not even

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[13] In the case of particles, statistical mechanics substitutes for this 'lore'.

[14] Stein offers this as an interpretation of a well known passage in one of Newton's unpublished papers where he insists that space '... has its own way of existing, which fits neither substances nor accidents.' (1962, p. 131) It is not a substance because it does not subsist 'absolutely of itself' and also does not have the characteristics of a substance. It is not an 'accident' because space does not 'inhere' in some subject, as accidents do. It is not nothing at all, because nothing can have no properties but we can conceive of space having properties (and in that sense it is close to substance). Thus, Newton concludes, 'Space is an affection of a thing *qua* thing' (1962, p. 136; Stein *op. cit.*, p. 396) and Stein understands this as the doctrine that '... *the fundamental constitution of the world* - its "basic lawful structure" - *involves the structure of space*, as something to which

conceptualized as a distinguishable thing to begin with, the question as to what is intrinsic amounts to no more than a request for which aspects of physical structure we denote as spatial. In particular, Stein insists that we must not let our ontology be driven by our mathematical representation[15]. That we separate the manifold from the metric and stress-energy tensors in our model of space-time does not justify any implication that the manifold somehow 'carries' or ontologically underpins the metric field. Of course, the field has to be defined 'on' the manifold to make mathematical sense but this should not be taken as carrying any extra metaphysical baggage (*ibid.*, p. 399).

Similarly, Hoefer's second alternative above can be understood as structuralist in this sense. Having rejected the characterization of space-time in terms of the manifold alone, with the attendant problems concerning the individuality of the associated points, he advocates a view of 'metric field substantivalism' in which the 'real representor' of space-time is the metric field:

> The division of the representors of the properties of space-time we see in describing space-time models as triples <M, g, T> does not indicate any deep dualism in the nature of space-time itself, on the view that I am describing here. Space-time has physical continuity, topology and metrical structure. (*op. cit.*, p. 24)

The focus on the metric field is justified in two ways: first there is an asymmetry between the metric and the manifold in that to give the former without the topology associated with the latter is still to describe part (albeit only the local part) of space-time, whereas to specify the manifold without the metric is not to give the space-time at all. Secondly, all the

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whatever may exist must have its appropriate relation.' (*ibid.*, p. 396; his emphasis) Space, then, is 'an aspect of the nature of thinghood' (*ibid.*) and placed in a modern setting, what this means is that 'Whatever exists of a physical nature ... must be appropriately related to a space-time manifold with a fundamental tensor-field satisfying the Einstein equations.' (*ibid.*, p. 397). Furthermore, Stein argues, this structuralist view is also expressed by Riemann in his talk of the 'real' or 'actual' that underlies space (*ibid.*)

[15] Cf. Maudlin (*op. cit.*, pp. 82-83) who similarly insists that we need to distinguish the representation from the object represented.

empirically useful and explanatory work within the theory of General Relativity is primarily done by the metric field. It is this which leads the substantivalist to claim that space-time cannot be given a relationist construal in the first place (cf. Maudlin *op. cit.*, p. 87).

As Hoefer has spelled out, this offers a way around the hole argument by denying the initial premise that space-time be identified with a manifold of points whose permutation has physical significance. However, the structural characterization brings out the point that it is not so much *primitive identity* that needs to be denied as *haecceitism* in general. Of course, this move entails the acceptance of Leibniz Equivalence and we may paraphrase Stein here and insist that we see no need to choose between the models. This is *not* to reduce this view to relationism if the latter is understood in the standard form in which the manifold is constructed from physical events (see Earman *op. cit.*, pp. 194-195); rather it is the relevant relations themselves that are accorded ontological status[16]. It may also offer a response to Grünbaum's request for a criterion of identity or distinctness for space-time points which can give physical meaning to the disidentifications involved in generating cover spaces. No such criterion is to be sought for because such disidentifications are not regarded as having any *physical* meaning in the first place, only a mathematical one. What is important, and what one should be a realist about on this view, are the relevant *structures* involved.

An appropriate epistemic context for such a view is provided by 'structural realism'. Structuralism has a long and honourable history in twentieth century physics and in its more recent incarnation (Worrall 1989; Ladyman 1998) it amounts to the claim that what we should be realist about are structures. In the case of quantum particles it has been argued that the above 'metaphysical underdetermination' between particles as non-individuals and particles as individuals can be broken by reconceptualising the particles in structural terms for which the issue of individuality simply does not arise (French and Ladyman forthcoming). What I take Stein to be suggesting is a similar reconceptualisation with regard to space-time. Just as we have to mathematically describe physical

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[16] It is a further interesting question whether this position differs significantly from Earman's attempt to do away with space-time points in terms of Leibniz algebras' (*op. cit.*, pp. 191-194).

objects using set theory, so we are pushed by our descriptive resources into a set-theoretic understanding of the underlying continuum, but, again just as in the case of particles, this does not mean we have to assign the mathematical points themselves any ontological significance. What are real are the relevant *structures* and it is to them that we should direct our philosophical attention. Having said that, there is clearly a great deal of serious work still to be done in spelling out the details of this programme, so perhaps we are not out of the hole just yet.

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