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CLAUSIUS' DISCOVERY OF THE FIRST TWO LAWS OF THERMODYNAMICS
A PARADIGM OF REASONING FROM INCONSISTENCIES[1]

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1. Aim and survey

In the middle of the nineteenth century, there were two incompatible approaches to the phenomena of heat and work. On the one hand, there was the view, mainly advocated by James Prescott Joule, that heat and work can be *converted* into each other; this view was supported by reliable experimental results. On the other hand, there was the theory of Sadi Carnot. This theory was highly successful, but was based on the idea that heat is a material substance ("calorique") that can neither be created nor destroyed. According to this theory, the production of work by a heat engine results from the mere *transfer* of heat from a hot to a cold body. Both approaches were reconciled by Rudolf Clausius in his *Ueber die bewegende Kraft der Wärme und die Gesetze, welche sich daraus für die Wärmelehre selbst ableiten lassen* (1850). The resulting theory can be considered as the foundation of modern thermodynamics. It is the first theory that contains the so-called First Law of Thermodynamics (conservation of energy) as well as (an early version of) the Second Law (entropy).

It is generally believed that the conflict between the two approaches

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[1] I am indebted to the late Yorgos Goudaroulis for introducing me to the history of thermodynamics, and to Diderik Batens, Jean Paul Van Bendegem and Erik Weber for comments on an earlier version.

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could be resolved by simply eliminating from Carnot's theory the view that heat is conserved. It is moreover accepted that Joule's view comes down to the First Law and that Carnot's theory incorporated the Second Law. In line with all this, Clausius' discovery is usually not regarded as creative (see, for instance, Mach 1896, Mendoza 1960b and 1961, Clark 1976, Smith 1990, and Psillos 1994). According to the common view, Clausius only had to resolve the conflict between the two approaches (which was quite trivial), and to combine the result.

The aim of this paper is twofold. First, I want to present a detailed reconstruction of the process by which Clausius arrived at his theory. Next, I want to argue that the standard account of Clausius' contribution to thermodynamics is mistaken: Clausius' theory is *not* a simple combination of (a contraction of) Carnot's theory and Joule's view.

In my 1993 (the companion to the present paper), I discuss some interesting mechanisms behind the process by which Clausius arrived at his theory: it required reasoning (from inconsistent premises) in order to detect specific contradictions and to resolve them. On the basis of these findings, I argue that Clausius' reasoning process cannot be accounted for by Classical Logic nor by (standard) nonmonotonic logics nor by monotonic paraconsistent logics. I also show that in order to understand Clausius' reasoning process we need an inconsistency-adaptive logic.[3]

In the present paper, I shall proceed as follows. After some preliminary remarks (section 2), I shall discuss the main shortcomings of the standard account of Clausius' contribution (section 3). In section 4, I shall present an analysis of the problem as it presented itself to Clausius. This will pave the way for a reconstruction of his discovery: section 5 will be devoted to the discovery of the First Law, section 6 to that of the Second Law. In section 7, I shall briefly discuss the central mechanisms behind Clausius' reasoning process. In section 8, I shall revisit the standard account, and argue that Clausius' discovery did involve creativity.

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[3] Inconsistency-adaptive logics were originally designed by Diderik Batens for the reconstruction of discovery processes that involve inconsistencies. An informal discussion of this type of logic can be found in Batens 1996; for the technical details, see Batens 1998; for an application to a specific historical example, see Meheus 199+.

2. Some preliminary remarks

The reconstruction of Clausius' contribution to thermodynamics which I present in this paper is based on his *Ueber die bewegende Kraft der Wärme und die Gesetze, welche sich daraus für die Wärmelehre selbst ableiten lassen* of 1850. This reconstruction is not a 'rational reconstruction' in the sense of Lakatos (my view of the way in which Clausius should have arrived at his theory), but an attempt to reconstruct, on the basis of an analysis of the original texts, as accurately as possible Clausius' reasoning process.

In this reconstruction, Clausius' discovery is viewed as a *problem solving process*. According to present-day methodologies of discovery, a problem consists of two components: a goal and a set of constraints — items of information that are relevant for the solution of the problem (Nickles 1980, 1981). In my terminology, the constraints may be of two kinds. *Limiting constraints* function as *conditions* on the solution (for instance, the requirement that the solution is consistent). *Constructive constraints* function as *premises* from which the solution can be *derived*. In accordance with this terminology, a problem is considered to be *well-defined* if a solution, that satisfies all the limiting constraints, can be derived from the set of constructive constraints.[4]

Given the complexity of the problem, it is implausible that Clausius reasoned exactly in the way that it will be presented here. However, the reconstruction reveals several interesting mechanisms (see section 7 and especially my 1993). Attempts to make the reconstruction more realistic will not undermine those mechanisms. Such attempts will only result in the discovery of more instances where Clausius arrived at new inconsistencies, more instances where he discovered new concepts, and more instances where he had to rework previously accepted findings.

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[4] I have recently refined the classification of constraints, see Meheus 1997 and Meheus & Batens 1996. In the new terminology, "constructive constraints" are called "relevant statements".

3. A criticism of the standard account

3.1 Clausius' discovery was not trivial

Most of the authors I consulted seem convinced that Clausius' theory resulted from the mere application of a simple heuristic: eliminate some parts of Carnot's theory in such a way that the outcome is consistent with Joule's view. According to Clark, for instance, Clausius resolved the conflict between Carnot's theory and Joule's ideas "*simply* by dropping Carnot's assumption that the quantity of heat remained undiminished when work was performed, while retaining the assumption that work results from the diminution of heat" (Clark 1976, p. 65, italics mine)[5]. According to Psillos, "[i]t was *easily* observed [...] that, if a hypothesis of Carnot's theory was overthrown [namely the idea that heat is conserved], the rest of the theory fitted perfectly well with Joule's important experimental findings. [...] The sound laws that had been established within the caloric theory were *readily* deduced and accounted for in the new theoretical framework of thermodynamics" (Psillos 1994, p. 186, italics mine).

If this holds true, then, obviously, Clausius' contribution did not involve much creativity. Any researcher, working at that time and familiar with the relevant results, could have made the discovery. However, this constitutes a serious underestimation of the problem as it presented itself to Clausius and his contemporaries. We know that solving the problem took several years. We also know, as even Clark admits, that William Thomson (later Lord Kelvin) considered the problem to be insurmountable (see, for instance, Clark 1976, p. 65). As Kelvin was one of the most competent researchers in the domain, it is hard to believe that he failed to apply a simple heuristic.

3.2 Clausius' theory was not a simple combination of existing ideas

According to the standard account, Clausius discovered nothing new: his theory is a *contraction* of Carnot's theory unified with some ideas of

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[5] Clark continues: "*the transition from Carnot to Clausius' theory involved simply dropping from Carnot's theory that part of it seen to be inconsistent with the conservation of energy*" (1976, p. 65).

Joule's. Smith (1990), for instance, gives a concise formulation of Carnot's theory in four propositions. As he presents it, Clausius' theory can be obtained by abandoning some of these and adding Joule's ideas to the remainder. Similarly for Mendoza (1960b) and Psillos (1994) who claim that the central concepts were already contained in Carnot's theory (the remainder being contained in Joule's ideas).[6] According to Mendoza, Carnot's "calorique" refers in most cases to the *entropy* of a body. According to Psillos, there is a sense in which "calorique" refers to the *internal energy* of a body.

The fact that different authors do not agree on how the terms of Carnot's and Clausius' theories correspond to each other suggests already that the relation between the two is not trivial. Moreover, if Clausius indeed did nothing else but selecting the true (or useful) part of Carnot's theory and to combine this with Joule's ideas, how then are we to understand that he considered it sufficiently interesting to give lengthy descriptions of the reasoning behind his results? And how can we make sense of the fact that he found it necessary to defend his ideas over and over again? Still in his 1863, for instance, he discusses at length various misunderstandings of his theory as well as fundamental objections to it.[7] It is hard to believe that Clausius' intended public needed an essay of eighty pages as well as additional clarifications to understand a *simple contraction* of a *known* theory. Besides, after Clausius formulated his theory, it took years before the Second Law was generally accepted. How can we make sense of this if we assume that a rough version of it was already present in Carnot's theory (which was by that time highly successful and generally accepted)?

4. An analysis of the problem as it presented itself to Clausius

One of the main problems with the standard account is that the

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[6] Mendoza and Psillos offer a 'realistic' interpretation of Carnot's theory. More specifically, they attempt to show that "calorique" is a referential term.

[7] The criticism was directed towards Clausius' fundamental principle that heat cannot flow by itself from a cold to a hot body. For the importance of this principle in the development of Clausius' theory, see section 6.

interpretation of both Carnot's theory and Joule's ideas is heavily influenced by modern thermodynamics. Most authors I consulted agree that Joule formulated the present-day principle of the conservation of energy (or at least something very close to it), and that Carnot discovered (something like) the present-day principle of increasing entropy.

This is devastating for a good understanding of the problem Clausius was trying to solve. Starting from such a whiggish reinterpretation, one unavoidably has the impression that Clausius faced a simple combination problem. In the following sections, I shall give a brief summary of those elements of early thermodynamics that are crucial to see Clausius' problem in the proper light.

In section 4.1, I present a brief introduction to the central assumptions of Carnot's theory. Joule's ideas are briefly discussed in section 4.2. Section 4.3 contains some comments on the incompatibility between the two approaches. In section 4.4, I discuss the main constraints of Clausius' problem.

For the sake of clarity and brevity, I introduce some terms that do not occur in the original texts. I shall take care, however, that Carnot's, Joule's and Clausius' ideas are reproduced as accurately as possible.

4.1 Carnot's theory and the fall of caloric

As I mentioned before, Carnot's theory was formulated within the caloric theory of heat. The latter has three important characteristics. (i) Heat is considered to be a material substance ("calorique") that can neither be consumed nor produced. (ii) A distinction is made between "total heat", "free heat" and "latent heat". Heat is called "free" if its presence in a body is discoverable by means of a thermometer; if this condition is not satisfied, it is called "latent"; the sum of the free heat and the latent heat is called "total heat". (iii) It is accepted that the free and latent heat of a body (and hence, its total heat) depend only on its actual condition, so that if all the other physical properties of the body are known, its free and latent heat are completely determined. In modern terms, we say that the three kinds of heat are regarded as *functions of state*. In line with this, Carnot accepted

C1 The amount of heat that has to be imparted to a body in order to effect a certain change is a function of state.

In accordance with his conception of heat, Carnot interpreted the working of a heat engine on analogy with the working of a water wheel: a heat engine receives heat from a hot body and delivers it to a colder body in the same way as a water wheel receives water from a high level which it emits at a lower level. In both cases, the production of work results from the 'fall' of a substance. Hence, Carnot accepted (pp. 9-11, pp. 6-7)[8]:

C2 The production of work by the agency of heat is always accompanied by the transfer of heat from a hotter to a colder body; the production of work by a heat engine is *not* due to the consumption of heat but results from the *mere* transfer of heat from a hot to a cold reservoir.

It is important to note that according to Carnot's theory, *every* difference in temperature can be exploited to produce work. However, when bodies of different temperature are in *immediate* contact with each other, there is a flow of heat that is *not* accompanied by the production of work. Consequently, Carnot accepted (p. 23, p. 13):

C3 Some processes result in a 'loss' of (potential) work.

To represent the action of a heat engine, Carnot devised a 'reversible engine cycle', later called a Carnot-cycle (see Figure 1). The device used is a cylinder, filled with a certain amount of gas and furnished with a piston. Further, there are two bodies (at different temperature) that function as *infinite* reservoirs of heat (the addition or subtraction of heat does not change their temperature). The cycle itself consists of four transformations in which the gas is expanded or compressed. To interpret these transformations, Carnot relies on the following assumption (pp. 29-32, p. 16):

C4 Whenever a gas is expanded (respectively, compressed), its temperature drops (respectively rises) *unless* a certain amount of heat is absorbed by it (respectively, given up by it).

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[8] The references to Carnot's paper are given by two numbers, referring to the relevant pages in the original French text (Carnot 1824) and in the standard English translation (Mendoza 1960a).

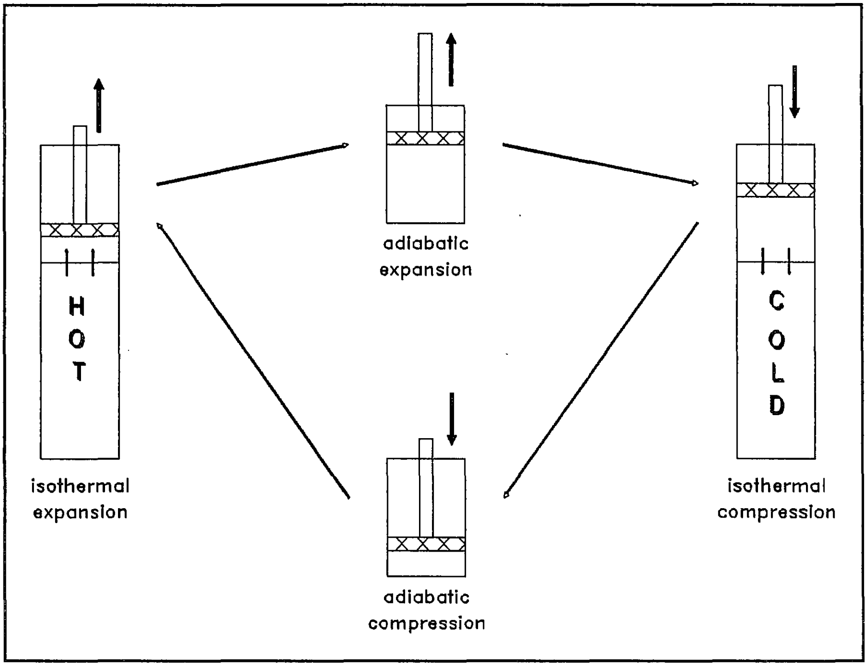

FIGURE 1

His interpretation of the four transformations is as follows (I call them by their present names): (i) isothermal expansion: the gas expands while the cylinder is in contact with the hot reservoir; the latter provides the heat needed to keep the temperature of the gas constant; the heat imparted to the gas during this operation is absorbed *in its entirety* by the gas (afterwards, all of it is present in the gas).

(ii) adiabatic expansion: no heat is added (the cylinder is thermally isolated); hence, the temperature of the gas drops.

(iii) isothermal compression: some heat flows from the cylinder into the cold reservoir, thus preventing the gas from heating up; the amount of heat that leaves the gas during this operation equals the amount of heat that was absorbed by it during the first operation.

(iv) adiabatic compression: no heat leaves the gas; hence, its temperature rises. The compression is continued until the temperature of the hot reservoir is reached.

At this point, the gas has returned to its original state, and the same operations can be repeated.

During the first two operations work is performed *by* the gas; during the last two work is performed *on* the gas. As the first two operations are performed at a higher pressure than the last two, there is a net gain of work. The operations can also be performed in the inverse sense. In that case, heat is transferred from the cold to the hot reservoir, and there is a net consumption of work. Hence, the net result of these four operations is given by the following assumptions (pp. 36-37, p. 19):

C5 Operating a reversible engine in the normal direction results in the production of work and the transfer of heat from the hot to the cold reservoir; operating a reversible engine in the reversed direction results in the consumption of work and the transfer of heat from the cold to the hot reservoir.

C6 The amount of heat absorbed by a reversible engine at one reservoir equals the amount of heat delivered at the other.

For a good understanding of Carnot's interpretation of the Carnot-cycle, also the following should be noted. (i) Both directions of a reversible engine annul each other: if the same amount of heat is transferred, the amount of work produced in the normal direction equals the amount of work consumed in the reversed direction. (ii) The amount of work a reversible engine produces with a given amount of heat is a function of the temperature of the reservoirs between which the heat is transferred. (iii) The operations of a reversible engine are arranged in such a way that bodies of different temperature are never in contact with each other. Hence, a reversible engine is an *ideal* engine: the amount of work produced by it is a theoretical *maximum* (no work is lost during the operations).

Relying upon these ideas and some auxiliary assumptions, Carnot derived the following theorem (pp. 21-22, pp. 11-12)[9]:

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[9] For a discussion of Carnot's argument, see my 199+.

C7 It is impossible to design an engine that produces *more* work than a reversible engine, while operating between the same temperatures and absorbing the same amount of heat (from a hot reservoir).

4.2 Joule's ideas and the mutual conversion of heat and work

Joule's first research concerned the design of electric motors. Early in his investigation he noticed the heating effect of a current (generated by some kind of dynamo). This discovery led him to the conviction that work may be *converted* into heat, and hence, that the production of heat results from the *consumption* of work. Joule also accepted the converse, namely that heat may be converted into work, and hence, that the production of work (by the agency of heat) results from the *consumption* of heat.

This view induced several remarkable experiments for the measurement of the 'mechanical equivalent of heat' (the amount of work that corresponds to one unit of heat).[10] Joule measured, for instance, the change in temperature of a gas when it is adiabatically compressed. Allegedly, he also established experimentally that the amount of work produced in an electro-magnetic machine is proportional to the heat extracted from the battery.[11] According to Joule's own account, the results of these experiments are well in line with the idea that heat and work may be converted into each other.

It is important to note that Joule viewed these conversions as mechanical processes which are perfectly *reversible*. Like most (if not all) adherents of the doctrine of mutual conversion, Joule put heat and work on the same level. "If they be convertible, they appear of the same value. If in doing work, energy was not consumed but only changed, it stood to reason that it might be changed back again, so that the work could be done over again. In other words, if all processes are purely mechanical processes — modes of motion — a supposition which very early forced itself with more or less clearness on the pioneers of the science of energy, they must be *reversible*: it must be possible to turn them round again, to undo what has been done, or to do what has been undone"

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[10] For a description of these experiments, see Joule 1845a and 1845b.

[11] By analogy, he claimed that the production of work by a heat engine results from the *consumption* of an equivalent amount of heat.

(Merz 1965, pp. 129, my italics).

So, from Joule's point of view, the heat consumed in producing work can be entirely recovered (it suffices to convert the work produced back into heat). Similarly, for the work consumed in producing heat.

It is clear, even from a common-sense view of nature, that this is impossible. But, as Merz observes, "it does not seem to have struck the earlier propounders of the doctrine of the equivalence and correlation of forces, such as Faraday, Mohr, Mayer, Grove — not even Joule and Helmholtz — that if neither matter or power is lost, the phenomena of loss and waste in nature and in human life remain unexplained" (Merz 1965, pp. 129-130).

If one takes into account all this, Joule's idea of mutual conversion can be expressed as follows:[12]

J1 Work and heat are *convertible* into each other: not only is the production of work by a heat engine due to the consumption of an equivalent amount of heat (and the production of heat due to the consumption of an equivalent amount of work), the heat (work) consumed during these conversions is entirely recoverable.

In view of **J1**, the following idea of Joule's seems self-evident:

J2 The total amount of work in the universe is *conserved*; no transformation can result in the loss of work.

4.3 The incompatibility between Carnot's theory and Joule's ideas

At the beginning of the 1840's, the caloric theory of heat was generally rejected in favour of a wave theory of heat.[13] However, **C2** was still considered as the base for thermodynamics. Starting from this 'fundamental axiom', Carnot's results as well as several new findings were derived. In the 1840's, also Joule's experimental results and ideas

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[12] I do not claim that the formulation is Joule's. I do claim, however, that it captures as accurately as possible the implicit assumptions underlying Joule's view.

[13] For a discussion of the wave theory of heat, see Brush 1986.

were accepted by a growing number of researchers.

However, from the joint adoption of Carnot's theory and Joule's ideas, at least two contradictions arise. As the reader may check, a first contradiction results from **C2** and **J1** and a second one results from **C3** and **J2**.

It was generally agreed upon that these contradictions caused serious difficulties. This should not surprise us: Carnot's theory was commonly considered as the *only* theory available for thermodynamic phenomena, and Joule's ideas were generally accepted as fundamental principles. Moreover, as Kelvin's reaction indicates (see section 3.1), resolving the inconsistencies that result from the joint adoption of Carnot's theory and Joule's ideas was far from trivial. Remember that Kelvin's reaction was not due to a lack of competence. It was related to the fact that Carnot's theory (when combined with Joule's ideas) leads to contradictions, the elimination of which is not at all obvious.

The reactions to the situation were diverse. Joule, having no commitments to the theoretical study of thermodynamic phenomena, simply rejected Carnot's theory as soon as he realized that it was incompatible with his experimental results as well as with some of his most fundamental tenets: "Believing that the power to destroy belongs to the Creator alone, I entirely coincide with Roget and Faraday in the opinion that any theory which, when carried out, demands the annihilation of force, is necessarily erroneous" (Joule 1845a, pp. 382-383).

Kelvin reacted in the opposite way. Unlike most of his contemporaries, Kelvin was sceptical of **J1**. He objected (quite correctly) that Joule's experimental findings demonstrated the conversion of work into heat but *not* the converse.[14] Moreover, he was convinced that, if one accepted Joule's ideas, thermodynamics had to be restarted from scratch. (Remember that **C2** formed the base of thermodynamics at that time, and that Joule's ideas are incompatible with it.) This is why he decided that, until a new foundation for thermodynamics was available, Joule's ideas had to be rejected in favour of Carnot's theory.

Clausius, who accepted a mechanical model of heat, reacted in a way

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[14] For an interesting discussion of the debate between Kelvin and Joule, see Smith & Wise 1989, pp. 302-316.

that was different from both Joule's and Kelvin's. On Clausius' model, heat is a kind of motion (it is related to the *vis viva* of molecules), and hence, it is subject to the laws of mechanics. Unlike Kelvin then, Clausius accepted **J1**, which was well in line with his mechanical model of heat. Unlike Joule, however, he realized that Carnot's theory was the only available theory for the study of thermodynamic phenomena. Because of this, he was reluctant to simply reject it. Instead, he tried to 'reconcile' it with his mechanical model of heat as well as with Joule's ideas.

4.4. Clausius' problem: its goal and its constraints

There can be no doubt about the *goal* Clausius was aiming at. Confronted with the incompatibility between Carnot's theory and Joule's ideas, he wanted to develop a theory that would satisfy the following *limiting constraints*: (i) it would be at least as powerful as Carnot's theory for the study of thermodynamic phenomena, (ii) it would be compatible with his mechanical model of heat, and (iii) it would be consistent.

What were the *constructive constraints*? To give an exhaustive list of all the (implicit and explicit) constructive constraints would be an impossible task. For the present purposes, however, we can restrict ourselves to four kinds: (i) Clausius' mechanical model of heat, (ii) some relevant experimental results, (iii) Joule's ideas and (iv) the central ideas of Carnot's theory. As Clausius, like most of his contemporaries, found it inconceivable that there could be a loss of work in Nature, it is most likely that he originally aimed at a theory from which **C3** would *not* follow.

Someone might object to including Carnot's theory among the constructive constraints, as it is clearly incompatible with Clausius' mechanical model of heat as well as with Joule's ideas. However, in order to arrive at a satisfying theory, both Carnot's theory and Joule's ideas were *needed*. Carnot's theory offers a detailed understanding of thermodynamic phenomena, but it is incompatible with Clausius' mechanical model of heat. Joule's ideas are in line with this model, but they are highly incomplete. Hardly anything interesting follows from them for the study of specific phenomena. The upshot is that Clausius, in order to arrive at a *consistent* and *complete* theory, had to make inferences from the *union* of Joule's ideas and Carnot's theory.

The reasoning process that led to the solution of Clausius' problem can be made comprehensible if one assumes that Clausius carefully investigated which parts of Carnot's theory and of Joule's ideas could be retained and which parts had to be modified or even rejected, but only *after* he had reinterpreted them in view of each other. This reinterpretation resulted in several changes to the set of constraints: some constraints were abandoned, others were modified, and new ones were added. Throughout these changes the original problem became gradually better defined, so that, finally, a solution was found.

The reader may have noticed that the original problem includes some 'personal' constraints — constraints that are typical of Clausius. The mechanical model of heat, for instance, was, at that time, not generally agreed upon by the relevant scientific community. We shall see that these and other personal constraints played a crucial role in redefining and solving the problem.

5. A reconstruction of the discovery of the First Law

Starting from Clausius' 1850, I shall now reconstruct the central steps of the discovery of the First Law. Evidently, the order in which Clausius presented his results does not necessarily reflect the way in which these results were obtained. I shall argue, however, that the order in which the results are presented corresponds most probably to the course of the original discovery process.

5.1 The analysis of changes in volume

The first chapter of Clausius' 1850 opens with the analysis of some processes in which a body is subject to changes in temperature and/or volume (pp. 7-10, pp. 112-115)[15]. It is plausible to assume that this analysis also formed the starting point of Clausius' reasoning process. First, changes in volume constitute paradigm cases for the conflict

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[15] When the references to Clausius' 1850 are given by two numbers, the first refers to original German text (Clausius 1850), the second to the standard English translation (Mendoza 1960a); when only one number is given, it refers to Mendoza's translation.

between Carnot's theory and Joule's ideas.[16] Next, examples like these are relatively easy to analyze. Finally, the analysis of changes in volume paves the way for a reinterpretation of the Carnot-cycle, which was one of the central elements of Carnot's theory.

Clausius' analysis of changes in volume led to a new interpretation of several terms of Carnot's theory. As I shall now show, these reinterpretations can be made comprehensible if one assumes that inferences were made from the union of Carnot's theory and Joule's ideas.

One of the examples Clausius discusses concerns the expansion of a gas. During the process considered, an amount of heat is imparted to an amount of gas in order to bring it to a higher temperature and volume. Let us analyze this example from the point of view of both Carnot's theory and Joule's ideas.[17]

As there is an increase in both the temperature and the volume of the gas, it follows from **C4** that

E1 at least some of the heat imparted to the gas is absorbed by it.

And, as some work is done by the expanding gas, one can infer from **J1** that

E2 at least some of the heat imparted to the gas is converted into work.

The combination of **E1** and **E2** already leads to a change in "heat imparted to the gas", for it entails that *part* of the heat imparted to the gas is absorbed by it, and that *part* of it is converted into work. But, there is more. It follows from **C1** that

E3 the amount of heat imparted to the gas is a function of state.

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[16] It is easy to see why: if a body changes its volume, work is produced or expended, and thus, according to **J1**, heat is consumed or produced; the latter contradicts Carnot's idea that heat is conserved.

[17] Precisely the same can be shown on the basis of the other examples Clausius discusses; the reasoning is only somewhat more complex.

However, as the work done is regulated by the pressure (which depends on the temperature), it makes a difference in the amount of work done, if the gas is first heated and then expanded, or if it is first expanded and then heated. In other words, depending on how the change takes place, a different amount of work will be done by the gas. From this observation together with **J1**, it follows that the amount of heat needed depends on how the transformations are performed. Hence, it follows that

E4 the amount of heat imparted to the gas is not a function of state.

The contradiction between **E3** and **E4** can easily be resolved by distinguishing different 'parts' in the heat imparted to the gas, and by reformulating the statements at issue in terms of these 'parts'. In line with **E1** and **E2**, it seems evident to distinguish between 'heat absorbed by the gas' and 'heat converted into work'. Thus, the contradiction can be resolved by reformulating **E3** and **E4** as follows

E3' the amount of heat absorbed by the gas is a function of state.

E4' the amount of heat converted into work is not a function of state.

Once this reformulation is obtained, it seems quite natural to relate the new terms to the distinction between free heat and latent heat. Thus, free heat can be interpreted as heat absorbed by a body, and latent heat as heat converted into work.

This is precisely what Clausius arrived at. Having analyzed the different examples of changes in volume, he partitions the heat imparted to a body in heat absorbed by the body and heat converted into work. The former he interprets as free heat; the latter as latent heat. Of these, he considers only the free heat as a function of state.

Note that there is an important difference between Carnot's and Clausius' interpretation of free heat and latent heat. According to Carnot's interpretation, *both* the free heat and the latent heat are present *in* the body under consideration. According to Clausius' interpretation, however, "we may consider only the former as really present in the vapour that has been formed. The latter is not only, as its name implies, *concealed* from our perception, but it is *nowhere present*; it is consumed during the changes in doing work." (p. 114).

This, at once, leads to a change in the concept of "total heat". The total heat, being the sum of the free and the latent heat, can no longer be conceived as the total amount of heat contained *in* a body. Instead, it has to be interpreted as the total amount of heat needed to bring a body from one state to another.[18]

Thus far, only Carnot's theory and Joule's ideas are used. Clausius' mechanical model of heat enabled him to refine the results obtained. Thus, Clausius related the free heat to the *vis viva* of the particles of the body. And, he distinguished between two kinds of work: internal work (related to overcoming the mutual attractions between the particles of a body) and external work (related to 'pushing back' an external pressure). In line with this, he partitioned the latent heat in internal and external. Of these, he regarded only the former as a function of state: it depends *only* on the body's initial and final state and *not* on the way in which the changes take place. Clausius used the symbol **U** to refer to the *sum* of the free heat and the internal latent heat (in accordance with the present-day terminology, I call this sum the "internal energy").

5.2 The analysis of the Carnot-cycle

Having (informally) analyzed some concrete examples of changes in volume, Clausius turns to a mathematical discussion of the subject. Here, he restricts himself to the consideration of *permanent gases*, which are, according to his own account, most clearly submitted to calculation (p. 10, p. 115).

Central in this mathematical discussion is the reinterpretation of the Carnot-cycle. There are at least two reasons why Clausius pays so much attention to this particular device. First, the Carnot-cycle formed one of the most important paradigmatic elements of Carnot's theory. Next, Clausius wanted to determine how much work a gas produces (or, which comes to the same, how much heat it consumes) when it undergoes

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[18] It is remarkable that Clausius in his 1850 holds to the original conception of the total heat — like Carnot, he defines the total heat as the total amount of heat contained *in* a body (pp. 7-8, p. 113). Only in a footnote added in 1864, he gives the new definition of total heat (Clausius 1864, p. 22). This supports the view that Carnot's theory formed an integral part of Clausius' original problem.

changes in temperature and/or volume. He realized, however, that there is a serious difficulty: "If any body changes its volume, mechanical work will in general be either produced or expended. It is, however, in most cases impossible to determine this exactly, since besides the *external* work there is generally an unknown amount of *internal* work done" (p. 117). As Clausius realized, this difficulty can be avoided if a Carnot-cycle is used (in which the gas returns to its original state). "In this case, if *internal* work is done in some of the changes, it is exactly compensated for in the others, and we may be sure that the *external* work, which remains over after the changes are completed, is all the work that has been done" (p. 117). Hence, "to determine the work produced by these changes [...], we need to direct our attention only to the *external* work" (p. 118).

Clausius does not discuss explicitly the way in which he arrived at a new understanding of the Carnot-cycle. However, given the common understanding of the cycle at that time and given the findings Clausius obtained from his informal analysis of changes in volume, it is not difficult to reconstruct the central steps in his reasoning. Let us begin with Clausius' reinterpretation of the four operations.

During the *isothermal expansion* as well as during the *adiabatic expansion* some work is produced by the gas. Hence, it follows from **J1** that during both operations an amount of heat is consumed. The difference is that only during the isothermal expansion heat is imparted to the gas. Hence, it follows from **C4** that only during this operation the temperature of the gas remains constant. The *isothermal compression* and the *adiabatic compression* are analogous to this: the expenditure of work is compensated by the production of an equivalent amount of heat; the temperature of the gas remains constant only during the *isothermal compression* (only here the heat produced is given up by the gas).

What about the net result of the cycle? From **C5** and **J1**, the following may be derived

R1 Operating a reversible engine in the normal direction results in the production of work, in the transfer of heat from the hot to the cold reservoir, *and* in the conversion of heat into work; operating it in the reversed direction results in the consumption of work, in the transfer of heat from the cold to the hot reservoir, *and* in the conversion of work into heat.

In accordance with this, Clausius interpreted a reversible engine as a device that (operating in the normal direction) absorbs an amount of heat from a hot reservoir, transfers *part* of it to a cold reservoir and converts *part* of it into work (hence, the transfer is 'partial' rather than 'total').[19] However, if the transfer is only partial, it follows that

R2 The amount of heat absorbed by a reversible engine at one reservoir is *not* equal to the amount of heat delivered at the other.

Thus, the reinterpretation of the reversible engine results in a new contradiction: **R2** contradicts **C6**. However, as Clausius must have realized, **C6** is related to one of Carnot's *implicit assumptions*, namely

R3 Heat can neither be consumed nor produced.

If the latter is abandoned, **C6** is no longer derivable (and hence, the contradiction resulting from **R2** and **C6** is resolved).

Having reinterpreted the Carnot-cycle, Clausius mathematically derived that the total amount of heat imparted to a gas, during an (infinitesimal) cyclic process, is equal to the increase of its internal energy U plus the heat consumed in doing external work (p. 17, p. 121). The total heat and the heat consumed in doing external work are expressed by inexact differentials, thus indicating that they are not functions of state, the internal energy by an exact differential. This actually constitutes the modern formulation of the First Law. A detailed discussion of its derivation is outside the scope of the present paper. Suffice it to say that the successive steps in Clausius' ingenious and rather complicated mathematical argument are incomprehensible if one does not take into account his former findings, namely (i) that the total amount of heat imparted to a body can be separated into several 'parts', and (ii) that only *some* of these are functions of state.

From the First Law, Clausius derived several important empirical results. One of them concerns the difference between the specific heat of a (permanent) gas at constant volume and at constant pressure (p. 26, p.

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[19] In line with his mechanical model of heat, Clausius interpreted this transfer as the *conversion* of heat at high temperature into heat at low temperature.

129). Although this empirical result was already derived by Carnot — in fact, it constituted one of the most important empirical successes of his theory — Clausius had to look for a *new* derivation (Carnot's derivation explicitly relies upon the idea of a *total* transfer). The fact that Clausius was able to derive the same result, even though he replaced the total transfer by a partial one, must have strengthened his confidence in his reinterpretation of the Carnot-cycle. Even more important is the derivation concerning the quotient of the two kinds of specific heat (p. 27, p. 130). Clausius discovered that this result, which formed an anomaly for Carnot's theory, can be derived from his reinterpretation of the Carnot-cycle. This result certainly must have put a premium on his reinterpretation.

5.3 The resolution of some inconsistencies and the discovery of new ones

We have seen in the previous section that Clausius added several new constraints to the original problem: the idea that there are several kinds of heat and that only some of them are functions of state, the idea of a *partial* transfer, ... These, however, are not the only changes to the problem. It is plausible to assume that Clausius gradually became confident enough of his reinterpretation to eliminate some parts of Carnot's theory — the empirical successes discussed in the previous section played an important role here. One of the constraints Clausius abandoned was **R3**. This principle, although fundamental to Carnot's theory, was incompatible with Clausius' mechanical model of heat. Moreover, the results obtained thus far indicate that there are no independent grounds for retaining **R3**: abandoning it does not make the reinterpreted theory empirically less successful than Carnot's theory (quite to the contrary).

Clausius also abandoned **C2**, but only after he had 'divided' it into several 'parts'; of these he retained

R4 The production of work by the agency of heat is always accompanied by the transfer of heat from a hotter to a colder body.

According to Clausius' own account, **R4** forms the essential part of **C2**: it is significantly verified by experience, and some of Carnot's most important results are based on it. Moreover, having reinterpreted the Carnot-cycle, Clausius realized that a transfer of heat does not necessarily conflict with the idea of an actual consumption of heat. As the example of the reinterpreted reversible engine shows, "it may very well be the case that at the same time a certain quantity of heat is consumed and another quantity transferred from a hotter to a colder body" (p. 112).

The abandonment of **R3** induces several other changes to Clausius' problem. Not only does **C6** no longer follow, the interpretation of several other assumptions of Carnot's theory changes. In **R4**, for instance, the meaning of "transfer" changes (partial instead of total); in **C7**, "reversible engine" changes (this term now refers to an engine that transfers only *part* of the heat it receives).

These changes led to the resolution of several inconsistencies (for instance, that between **C2** and **J1**). However, the total set of constraints is still inconsistent. More specifically, the contradiction resulting from **C3** and **J2** is not yet resolved. There is something more, however.

It is likely that Clausius not only reinterpreted Carnot's theory in the light of Joule's ideas but that he also analyzed the latter in view of the former. At some point then, he must have realized the following. Suppose that a body is heated by means of friction. According to **J1**, the work thus expended can be entirely recovered — it suffices to convert the heat produced back into work. But, if **C7** holds true, it is impossible to design an engine that produces more work than a reversible engine. Given the new interpretation of the reversible engine, this entails that, if all the heat produced is absorbed by even the most efficient engine, only *part* of it will be converted into work (the remainder being *transferred* to a second reservoir). This entails, in contradiction to **J1**,

R5 There are cases in which the work consumed in producing an amount of heat is not entirely recoverable.

It also entails, in line with **C3** but in contradiction to **J2**, that some processes result in the loss of work.

These new difficulties must have been considered by Clausius as extremely serious. When he started working on the problem, he considered **J1** as well as **J2** as fundamental and indisputable principles. On the other hand, **C7** formed one of the central elements of Carnot's theory from which several important results were derived.

As a reaction to these difficulties, Clausius tried to find out whether **C7** still follows from the reinterpreted theory. We shall see in the next section that this attempt led not only to the discovery of the Second Law, but also to a modification of **J1**.

6. A reconstruction of the discovery of the Second Law

6.1 A new derivation of C7

In my 1993 and 199+, I give a detailed reconstruction of the way in which Clausius arrived at a new derivation of **C7**. This reconstruction is based on the argument Clausius presents in his 1850, and on some comments he made on this particular stage in his reasoning process (1863, p. 313).

As some readers may know, Carnot's derivation of **C7** is based on the impossibility of a perpetual motion machine (if a reversible engine is combined with an engine that produces *more* work while receiving the *same* amount of heat, one obtains a perpetual motion machine). Apparently, Clausius first tried to 'replicate' Carnot's argument from the point of view of Joule's ideas. He decided, however, that **C7** cannot be derived in this particular way. As soon as one accepts that the production of work by a heat engine requires the consumption of an equivalent amount of heat, one cannot possibly arrange two heat engines in such a way that the result is a perpetual motion machine. (If the combined engine produces work, an amount of heat is consumed, and hence, it is not the case that work is produced 'out of nothing' — see also my 1993 and 199+).

At this particular point, Clausius made an interesting move. Having discovered that Carnot's set-up (combination of two engines that produce a *different* amount of work, while receiving the *same* amount of heat) does not violate a fundamental principle, he tried out a *transformation* of it (combination of two engines that produce the *same* amount of work, while receiving a *different* amount of heat). As Clausius showed in his 1850, the new set-up results in the transfer of heat from a *cold* to a *hot* reservoir, *without* any expenditure of work or any other change. This result contradicts a principle Clausius considered absolutely fundamental, namely (p. 32, p. 134):

R6 It is impossible, without the expenditure of work, to transfer heat from a cold to a hot body.

Having obtained this contradiction, Clausius derived **C7** (by *reductio ad absurdum*).

How did Clausius arrive at **R6**? This question, as far as I know, cannot be answered with certainty. That heat (caloric) always tends to an equilibrium, is an important aspect of Carnot's theory. However, for all I know, it was not explicitly mentioned in discussions of Carnot's theory at that time. So, I do not know whether this principle originally belonged to the (explicit) constructive constraints of Clausius' problem, or, that he 'discovered' it while he was trying to find a new derivation for **C7**. In any case, as soon as Clausius became 'aware' of this principle, he assigned the highest preference to it. In a note added in 1864, he even claims that it is as fundamental as the principle that neither heat nor work can be created 'out of nothing' (Clausius 1864, p. 55).

As I discuss in detail in my 1993, the new derivation of **C7** had an unexpected side-effect: it showed that the adoption of **J1**, or more precisely of one of its 'parts', namely

R7 The work consumed in producing heat is entirely recoverable.

would make it impossible to arrive at a consistent theory. As I explain in my 1993, this provided good (internal) grounds for abandoning **R7** in favour of **R5**, and **J2** in favour of **C3**. Thus, ideas were abandoned which Clausius originally considered as highly plausible in favour of ideas he originally considered as highly implausible.

Having found a new argument for **C7**, Clausius used it (quite successfully) in the derivation of important empirical results. One of the empirical successes concerns the derivation of a theoretical value for the 'mechanical equivalent of heat': *the work equivalent of the unit of heat is the lifting of something over 400 kilograms to the height of 1^m* (p. 151). Clausius compared this result with the results Joule obtained from different experiments and concluded that the agreement formed a strong confirmation for **C7** (p. 152).

6.2 All inconsistencies resolved

The second part of Clausius' reasoning process resulted in the acceptance of some new constraints (**R6**, for instance). It also resulted in the abandonment of central elements of Joule's ideas. As I show in detail in my 1993, these changes were based on a logical analysis of the relevant constraints and led to the resolution of all the remaining inconsistencies.

It is remarkable that Clausius, in his 1850, pays little attention to the far-reaching consequences of his arguments. Although he clearly adopts **C7**, the idea that the work consumed in producing heat is not entirely recoverable, and that there may be a loss of work is only implicitly dealt with. Most probably, this has to be understood as a rhetorical move. In order to find acceptance for his theory, Clausius highlights the empirical successes of it, rather than the dramatic consequences for generally accepted assumptions.

Well-informed readers recognized almost immediately the full significance of Clausius' arguments. Kelvin's reaction is illuminating here. Having assimilated Clausius' theory, he gave his own formulation of thermodynamics (Thomson 1852 [1851]). In this formulation (which relies upon Clausius'), Kelvin explicitly deals with the idea that the amount of (useful) work in the universe is not conserved. This idea will also become one of the central themes in Clausius' later writings. There, it will give rise to the concept of entropy and to the present-day formulation of the Second Law (see, for instance, Clausius 1863).

7. Some mechanisms behind Clausius' reasoning process

It is typical of Clausius' reasoning process that inferences are made from inconsistent constraints. These inferences result in the discovery of several contradictions. The reinterpretation of changes in volume, for instance, results in the contradiction between **E3** and **E4**. As I discuss in more detail in my 1993, at least two mechanisms play a role in the resolution of these contradictions.

First, there is the logical analysis of the relevant constraints. This analysis leads to the discovery of relations between contradictions — as, for instance, in the case of the contradiction between **R2** and **C6** (see section 5.2). It also motivates the choice which 'halves' of inconsistencies should be abandoned (as, for instance, in the case of the contradiction between **R5** and **R7**).[20]

Next, there is the discovery of new concepts. The importance of this mechanism for Clausius' reasoning process can hardly be overestimated. At several points, Clausius discovered, on the basis of an analysis of *inconsistent* constraints, new concepts which helped in the formulation of a *consistent* alternative. Let me illustrate this with Clausius' reinterpretation of changes in volume — see section 5.1).[21]

The analysis of changes in volume led to a very interesting conceptual change — a specific differentiation in the concept of heat. This conceptual change, which formed a key element in Clausius' solution, did *not* result from some obscure mechanism, but can be understood as the result of straightforward *reasoning* from two incompatible interpretations of the same phenomenon. It is especially fascinating that it is impossible to arrive at this specific differentiation by analyzing Carnot's theory and Joule's ideas *independently* of each other. Carnot partitions the heat imparted to a body in free heat and latent heat. However, both the free heat and the latent heat are present *in* the body. Joule does not even consider a partitioning of the heat imparted to a body.

Things change, however, if inferences are made from the *union* of Carnot's theory and Joule's ideas. In that case, one is able to derive that *some* of the heat imparted to an expanding body is *absorbed* by it (thus affecting its temperature), *and* that *some* of it is *converted* into external work. This differentiation does not only lead to a better understanding of thermodynamic processes, it also enables one to resolve a central contradiction, namely that the heat imparted to a body is and is not a function of state (see section 5.1).

Someone might object that this specific differentiation is entirely compatible with Joule's ideas. This certainly holds true. Joule by no means excludes that there are several kinds of heat and that only a *portion* of the heat imparted to an expanding body is converted into (external) work (the remainder being absorbed by the body and/or converted into internal work). However, this does not undermine my claim that this specific conceptual change cannot be obtained on the basis

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[20] For a detailed discussion of this first mechanism, and for more examples, see my 1993.

[21] Other examples can be found in my 1993.

of Joule's ideas alone: they do not allow one to determine if (and how) the heat imparted to a body should be partitioned.

8. The standard account revisited

In the previous sections, I have shown that the problem as it presented itself to Clausius did not have a standard solution. Not only was it far from trivial to localize and resolve the relevant contradictions, the final solution turned out to be very different from what was originally intended. Remember that Clausius eventually rejected ideas which he initially considered as unquestionable — for instance, the idea that the work consumed in producing heat is recoverable (see section 6.1). Because of this, no researcher at that time could have predicted the final solution. It was only through a thorough analysis of the relevant constraints that it became clear that the acceptance of **J2** and **R7** prevented the formulation of a consistent theory. Note also that in this analysis, Clausius relied heavily on personal constraints — for instance, the mechanical model of heat (central for the reinterpretation of the Carnot-cycle) and the principle that heat cannot be transferred from a cold to a hot body without the expenditure of work (crucial for the new derivation of **C7**). Hence, it is not by chance that Joule or Kelvin missed the solution Clausius found; given their interpretation of the problem at that particular moment, *they could not have arrived at Clausius' solution*. The upshot is that, contrary to the standard account, Clausius' contribution to thermodynamics did involve creativity.[22]

I have also shown that Clausius' theory cannot possibly be characterized as a simple combination of Carnot's theory and Joule's ideas. Not only were central parts of *both* abandoned, the parts which were retained acquired a new meaning. As an example, one may think of **C2**. According to Carnot's interpretation, the production of work requires a *total* transfer of an amount of heat from a hot to a cold body. According to Clausius' interpretation, it requires a *partial* transfer. Similarly for **C7**. According to Carnot, a reversible engine simply

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[22] For the importance of 'personal constraints' in creativity, see my 1997 and also Batens & Meheus 1996.

transfers caloric from one reservoir to another. According to Clausius, a reversible engine is characterized by the conversion of heat *and* by the transfer of heat. It should also be noted that Clausius had to find *new* derivations for all the results Carnot derived from his theory — one should think here not only of **C7** (see section 6.1), but also of the empirical findings Carnot derived (see, for instance, the last paragraph of section 5.2).

One aspect of the reconstruction deserves special attention. I have shown that in order to arrive at his theory, Clausius had to make inferences from the *union* of Carnot's theory and Joule's ideas. This holds especially true for the First Law. Contrary to what is commonly accepted, this law was not obtained on the basis of Joule's ideas alone. The reason for this is that Joule's idea of mutual conversion was related to the idea that *work* is conserved (see section 4.2). It was *only* when Joule's ideas were confronted with (a reinterpreted version of) Carnot's theory that Clausius was able to detach the idea of mutual conversion from the idea that work is conserved (see section 6.1).

As I mentioned already, it took a long time before Clausius' theory was accepted by the relevant scientific community. From the point of view of the standard account of Clausius' contribution, this is incomprehensible. The reaction of Clausius' contemporaries makes sense, however, if one realizes that Clausius' theory departed not only from particular parts of Carnot's theory, but also, and much more importantly, from Joule's assumptions concerning the reversibility of physical processes and the conservation of work. The modifications to Carnot's theory (resulting from the abandonment of **R3**) were quite easily accepted. The modifications to Joule's ideas, however, were a lot harder to accept.

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