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TRANSITION AND CONTRADICTION

John R. McKie

I

Due largely to the contemporary interest in paraconsistent logic, the fourteenth-century *incipit/desinit* (begins/ceases) problem has recently been revived in new form.[1] Like Zeno's paradoxes of motion, to which it is closely related, this problem is easier to state than to resolve: is an object in motion or at rest *at* the instant of change? Consider the following possible answers to this question:

(1) The object is at rest but not in motion.
(2) The object is in motion but not at rest.
(3) The object is neither at rest nor in motion.
(4) The object is both at rest and in motion.

Unfortunately, none of these alternatives is very appealing. Briefly, alternatives (1) and (2) are unattractive because they appear arbitrary: there is no apparent reason (physical, mathematical, or metaphysical) for preferring one to the other. Alternative (4) is unattractive because it entails that motion is contradictory, given the intuitively plausible assumption that rest and motion are mutually exclusive states of an object. And alternative (3) is unattractive because it entails that the universe is indeterminate at the instant of change. (Indeed, given the assumption that rest and motion are jointly exhaustive, plus some other elementary logical machinery, this alternative also entails that motion is contradictory.)

It is important to realize that this problem remains even when full information regarding position with respect to time is given. Imagine that up to time t_1 an object is at place x_1, and occupies the positions x_n at time

t_1 and thereafter: $f(t_n) = x_1$ (for $t_n < t_1$) and $f(t_n) = x_n$ (for $t_n \geq t_1$). Here we have full information regarding position with repect to time, and therefore all the relevant information about the kinematic state of the object (if facts about motion and rest supervene on facts about position over time). In particular, we know that $f(t_1) = x_1$. But the question still remains whether the object is moving or at rest at t_1. As Jackson and Pargetter point out, it would be unwarranted to conclude that since $f(t_n) = x_n$ (for $t_n \geq t_1$) the object is moving at time t_1, because the following specification puts the object in precisely the same positions at precisely the same times: $f(t_n) = x_1$ (for $t_n \leq t_1$) and $f(t_n) = x_n$ (for $t_n > t_1$). By parallel reasoning this gives us equal warrant for saying that the object is at rest at t_1.[2]

Typically, attempts to deal with this problem involve trying to make options (1)-(4) less unattractive. For example, pointing out the prevalence of stipulation in mathematics might make alternatives (1) or (2) more palatable; distinguishing between metaphysical indeterminacy and epistemological opacity might make alternative (3) more attractive; or developments in paraconsistent logic might make alternative (4) more attractive.[3] One approach to the problem that has been largely neglected, however, involves the unconventional strategy of rejecting the continuity of space and time altogether. It is the intelligibility of this approach — in particular, the attendant idea of discrete space and time — that I want to explore in the present paper.[4]

II

Intervals of discrete space and time are ultimately divisible into degenerate subintervals which are themselves positively extended but not further divisible; these degenerate subintervals are sometimes called *hodons* and *chronons* respectively. Leaving aside complications, and idealizing greatly, the solution to the *incipit/desinit* problem this affords is simple: at one chronon of time t_n a moving object is at hodon x_n and at the next chronon of time t_{n+1} it is at hodon x_{n+1}. (Whereas the instants of continuous time, being dense, are successively but not consecutively ordered, the minimal subintervals of discrete time are both successively and *consecutively* ordered — they exhibit *nextness*.) But since a chronon of time is *ex hypothesi* indivisible it has no boundaries (and no earlier or later within itself) and therefore no question arises of the state of an object *at* the "instant" of change.

But there are real difficulties in excluding from this schema assumptions properly associated only with continuous space and time, as the following comments reveal:

> The atomicity of a body in space, of an event in time is conceivable, but how are we to conceive of the atomicity of space and time themselves? To ascribe a duration, however small, to a chronon, is to limit the chronon to two successive instants, thus reintroducing the infinitesimal element we wanted to exclude.[5]

Hamblin expressed the same doubts:

> There are difficulties in the concept of a time-scale without instants, for there must surely be time intervals, and intervals appear to require instants as their ends even if they are not, in fact, actually made up of continuous assemblages of them.[6]

However, to insist that a chronon of time *must* separate two instants or that a hodon of space *must* separate two points simply reveals an inability to relinquish the ideas associated with continuous space and time. If hodons and chronons are indivisible they have no boundaries. ("That which has no parts can have no extremity."[7]) There cannot be an earlier part and a later part (even an earlier bounding instant and a later bounding instant) of an *indivisible* chronon. Nor can hodons have diameters, hypotenuses, radii, or any other intrinsic metrical properties or relations, for this would again entail their divisibility. (Hence, hodons cannot be square, or circular, or triangular. Shape must be thought of as a property of *aggregates* of hodons in discrete space (and, derivatively, the objects which occupy them), not of individual hodons.) Although plausible intuitively, the assumption that geometrical properties are invariant with respect to changes of scale — in particular, that small intervals, no matter how small, are divisible just as larger ones are — must be resisted in the case of discrete space and time.

III

Hamblin notes that abandoning the continuity of space and time does not *ipso facto* require the adoption of discreteness — *i.e.* the adoption of an elementary constant of length or duration. On the account he develops, intervals of space and time, while infinitely divisible, are *neither* constituted of points and instants *nor* of ultimate subintervals, but merely consist of subintervals of ever decreasing size.[8] On this view, all spatial and temporal intervals have proper parts, but there are no points, instants, or ultimate subintervals (hodons or chronons).[9]

Although this does away with continuity, it is not strictly a discrete solution, for it does not incorporate a constant of length or duration. How might we determine, for example, the area of a circle in a discrete space incorporating such a constant of length? Being mindful of the potentially misleading nature of pictorial representations, we might imagine two-dimensional discrete space as an infinite chess board, each individual square being a hodon of unit area. Then the area of a circle of a given radius, r, can be associated with the number, N, of individual squares it encompasses:

$$C(r) = N_{C(r)}.$$

Imagine the squares are pressure-sensitive, and we lay a disk of radius r on the board. If every square containing some point in the disk is turned on, and every point in the disk is in some square which is turned on, the difference between C(r), the combined area of the squares turned on by the disk, and the classical Euclidean area of the circle, πr^2, is at most equal to the area of the squares that fall on the boundary of the disk B(r):

$$C(r) - \pi r^2 \leq B(r).$$

Since the maximum distance between any two points of a square is $\sqrt{2}$ (assuming that the tiles are of unit area, and using Euclidean geometry as a useful idealization), we can be sure that all of the squares that fall on the boundary of the disk are contained within an annulus of width $2\sqrt{2}$. This annulus has an inside radius of $r - \sqrt{2}$ and an outside radius of $r + \sqrt{2}$ and its area is:

$$A(r) = ((r+\sqrt{2})^2 - (r-\sqrt{2})^2)\pi = 4\sqrt{2}\pi r.$$

But the combined area of the squares that fall on the boundary of the disk is less than the area of the annulus. Thus:

$$C(r) - \pi r^2 < 4\sqrt{2}\pi r.$$

If we now assume that the radius r is large compared to the size of the individual squares we may assert the following approximate equality:

$$C(r) \approx \pi r^2.$$

In other words, the area of a circle in discrete space is approximately equal to the area of a circle in continuous space. It is not difficult to extend this type of reasoning to other shapes, familiar and arbitrary, in *n*-dimensional space.[10]

This does not provide us with a discrete substitute for Euclidean geometry, of course, since it freely utilizes Euclidean geometry in establishing the relevant approximations. This method can only aid the imagination, and indicate the possibility of a correlation between Euclidean geometry and discrete space which preserves, approximately, some of the important metrical relationships of the former. Van Bendegem has also shown how a tile model of discrete space can accommodate the Pythagorean theorem to within a tolerable degree of approximation.[11]

Theoretical considerations have prompted some physicists to posit a size of approximately 10^{-13} cm. for space quanta, and 10^{-24} sec. for time quanta. The classical electron radius is 10^{-13} cm., as is the radius of the proton, and the distance of strong nuclear interactions is of the same order of magnitude.[12] Although the fact that the figure 10^{-13} cm. appears repeatedly in the theoretical representation of matter, energy, radiation, and so on, does not prove anything about the structure of spacetime, it is nevertheless natural to look for an explanation for the recurrence of a physical magnitude, and the idea that this number (or another with comparable credentials) might indicate some type of limitation inherent in spacetime itself cannot be rejected lightly.

For example, Abramenko has suggested that there may be a connection between the discrete structure of spacetime and Planck's quantum of action. He notes that if we imagine a quantum of spacetime of the dimensions 10^{-13} cm. and 10^{-24} sec. filled with matter of nuclear density, the energy content will be of the same order of magnitude as Planck's quantum of action. "An idea suggests itself that the elementary quantum of action has something to do with discrete structure of the world, and with the elementary and irreducible quantum of spacetime."[13] The connection suggested here between Planck's quantum of action and discrete spacetime raises the possibility that all elementary particles and all quantum phenomena may be explainable in some way by appeal to the discrete structure of spacetime, and this in turn raises the possibility that some of the diversity of present-day particle physics might be eliminated under the unifying influence of a discrete conception of the spacetime manifold.[14] Current work on quantization of the gravitational field is the most recent outcome of this trend.[15]

Several possible advantages (all highly speculative of course) might be cited for discrete space and time. First, as just noted, discrete spacetime offers the possibility of a more coherent account of a diversity of phenomena to do with measure, duration, energy, and so forth in the quantum domain. In particular, discrete spacetime is often cited as a possible way of avoiding re-normalization problems, and the infinite self-energy of the point-electron. For example, according to Hellund and Tanaka "it seems plausible that a theory of elementary particles based on a universal constant of the dimensions of length will be free of divergence difficulties, removing one of the most important obstacles to the progress of field theories."[16] Second, as Hamblin noted, continuity seems a richer structure than we need for a description of the physical world: "The red book on my desk could turn green for half a second or half a century but it could not turn green temporarily and durationlessly at the stroke of twelve, remaining red at all times earlier and later. This being so, the time-continuum, modeled on the real numbers, is richer than we need for the modeling of empirical reality."[17] Finally, discrete spacetime provides the possibility of a reconciliation between general relativity theory and quantum mechanics via quantization of the gravitational field. "There is still the mystery of the Planck length, $(Gh/c3)^{1/2} \sim 10^{-33}$ cm, at which the structure of space-time and quantum effects would become inextricably intertwined. The feeling that, at this distance, some profound change in our conceptual understanding of the physical world would be necessary, motivates and inspires much of the research into quantum gravity."[18]

IV

But significant problems also face theoretical speculation on discrete space and time. For example, Priest suggests that postulating infinitely subdivisible intervals in the absence of hodons and chronons, as Hamblin suggests, might lead to contradiction. Imagine a property *a* which holds at an interval *X*. Just as saying that the sun shone on a certain day does not imply that it shone during every part of the day, so there is nothing incompatible in saying that every subinterval of *X* where *a* holds has a (proper) subinterval where $\sim a$ holds (and *vice versa*). But this raises the embarrassing question: "What holds at this interval? What could it be but $(a \mathbin{\&} \sim a)$?"[19]

The mistake in this argument, however, is that talk of what holds *at* an interval is ambiguous. Since instants have no parts, there is no relevant distinction between some property holding "throughout" an instant or during some "part" of that instant. But the situation is different with infinitely subdivisible intervals. To say that a holds *at* an interval can either mean that a holds *throughout* that interval (von Wright calls this a *univocal* characterization of the world during that interval with respect to a[20]) or that a holds for certain periods during that interval. If Priest means that a holds throughout the interval X, then this is incompatible with his claim that every subinterval where a holds has a subinterval where $\sim a$ holds (or else he has already built a contradiction into his premises). On the other hand, if he means that a holds for certain periods during the interval X, then this is quite compatible with $\sim a$ also holding for certain periods during the interval X. The same reasoning holds *mutatis mutandis* for any subinterval of X. Of course, since *every* subinterval of X where a holds has a (proper) subinterval where $\sim a$ holds (and *vice versa*), we never arrive at a subinterval *throughout* which $\sim a$ holds — *i.e.* during which a does not hold — (or *vice versa*); but by the same token, we never arrive at a subinterval during which both a and $\sim a$ hold but which cannot be further subdivided — we never arrive at a subinterval throughout which both a and $\sim a$ hold — so no contradiction results. We may, if we like, call this "a contradiction of a kind", but it is not "a contradiction of the paradigm type."[21]

Priest also observes that on the discrete account there is no intrinsic difference, at any particular chronon, between an object in a state of rest and an object in a state of motion. Rest and motion are relational properties, and can only be determined by appeal to neighbouring times. But according to Priest, "this conception of motion jars against our intuitive notion of motion as a genuine flux."[22] According to Priest's neo-Hegelian account, the intrinsic difference between an object at rest and an object in motion is that "the instantaneous moving state is a contradictory state, whilst the instantaneous rest state is not."[23] What is sufficient for a body to be in motion at a particular time "is for it both to occupy and not to occupy a certain place."[24]

Several comments seem relevant here. First, even granting that the law of non-contradiction is not an unassailable *a priori* truth, many will find this too high a price to pay for a non-relational state of motion. *Prima facie*, rejecting the idea of an intrinsic state of motion offends less against our intuitions than rejecting the law of non-contradiction. Second, concerning the continuous account of motion (and a corresponding argument applies in the discrete case) Priest comments: "If God were to take temporal slices of an object at rest in different places, and string them together in a continuous fashion, he would not have made the object move."[25] However, this begs the question. What would God have left out? The intrinsic "quality" of motion? To assume that there is more to motion than being at the appropriate places at the appropriate times is to beg the question at issue. Third, according to *The Shorter Oxford English Dictionary*, "flux" means "the action of flowing," "a continuous succession of changes," "the flowing in of the tide." If this captures "our intuitive notion of motion," it is not something that can happen at an indivisible chronon (or instant) of time, Priest to the contrary notwithstanding.

Priest and Mortensen also independently raise legitimate doubts concerning the role of the theory of differential equations in physical theory.[26] In the absence of its demonstrated feasibility, just how confident can we be that an alternative of the kind Hamblin envisages, or any other theory of discrete space and time, will be able to account for the same total body of empirical findings?

Several comments also seem relevant here. First, we should not interpret too negatively the fact that a discrete reconstruction of (or alternative to) differential and integral calculus adequate to capture the empirical facts of motion (velocity, acceleration, ...) has not been developed. After all, the modern mathematical theory of continuity itself is less than one hundred years old, and the dispute between the defenders of continuity (or infinite divisibility) and the defenders of discreteness has been raging for over two thousand years. Second, it must be kept in mind that quantities which are known to be discrete are repeatedly denoted in modern physics by differentials despite the fact that differentiability presupposes continuity. The law of radioactive decay, for instance, is given in terms of a continuous exponential function, even though radioactive decay involves the emission of discrete quantities of energy. Other examples could be given from statistical mechanics and electrodynamics.[27] Third, due to the mathematical difficulties attending the solution of, for example, finite-difference equations, it would be admittedly unpalatable to recast all equations of motion in the form of difference equations instead of differential equations. But the acceptance of spatio-temporal discontinuity in physics does not preclude us from applying mathematical concepts involving spatio-temporal continuity in our calculations. (It should also be noted that there is nothing incoherent in the idea of a space which is metrically discrete but topologically continuous.) Fourth, a mathematical formalism which faithfully reflects a discrete spacetime (which introduces new primitives, defines structures on them, and so forth) is not necessarily going to saddle us with the use of difference equations, or be difficult to treat, particularly in the future.[28]

V

A paradoxical conclusion Russell thought derivable from the discreteness hypothesis for space and time is that all objects must move with the same velocity. A moving object cannot occupy the same hodon for two consecutive chronons for then it would not be moving. But neither can it, from one chronon to the next, travel further than from one hodon to the next, for this would require the object to occupy the intervening hodons at no time at all. Thus, all moving objects in discrete space and time must pass from one hodon at one chronon to the next hodon at the next chronon: "there will be just one perfectly definite velocity with which all motions must take place: no motion can be faster than this, and no motion can be slower."[29]

However, by occupying hodons for more than one chronon the velocity of an object can be retarded to any velocity whatever while continuing to *appear* perfectly smooth and uninterrupted in its motion, since the extremely brief times for which a "smoothly" moving object pauses (10^{-24} sec. = a million millionth part of a million millionth part of a second) will not be perceivable as periods of rest at all.

More plausibly, it might be thought that Russell's argument does establish the existence of a *maximum* velocity with which all objects must move in discrete space and time: at the rate of one hodon per chronon. In this connection it is relevant to note that the figure 10^{-24} sec. represents the time it would take a light beam (*in vacuo*) to traverse a distance of 10^{-13} cm. The maximum velocity of one space quantum per time quantum is thus approximately the speed of light, and the duration of the chronon may be interpreted as the shortest period in which causal interaction can occur. So it should not be surprising if the maximum speed in discrete spacetime was the speed of light. Furthermore, Russell's argument assumes that it is not possible for an object to traverse a number of distinct hodons in only one chronon of time. We are not bound to accept this assumption in the absence of further evidence. Perhaps tachyons move in this way.

An obvious difficulty confronting discrete spacetime concerns its compatibility with relativity theory. Relativity theory is hostile to the introduction of a constant of length because it seems to single out privileged directions in space, and because it is invariant under Lorentz transformations. In a letter to Ilse Rosenthal-Schneider in 1953 Einstein commented on Heisenberg's postulation of a smallest unit of length in the following way:

> It is difficult to think of carrying through relativity theory in any other way than on the basis of the continuous field which for its part does not admit any singularities.... The endeavor to introduce something like a lowest length cannot, in my opinion, be carried through consistently at all. In my view it is nothing more than the attempt at a lame compromise between point mechanics and field theory.[30]

However, a discrete spacetime method has been used by Henning to evaluate the spacetime uncertainties for relativistic particles; E.L. Hill has gone some way toward showing that discrete spacetime need not do utter violence to the invariance under Lorentz transformations; and curved space approaches to space quantization have been proposed by Yang and Flint.[31] While the compatibility of discrete spacetime with relativity theory is a significant problem, these studies suggest that it is perhaps not insurmountable.

I have not suggested in this paper that the discrete approach to spacetime is preferable to the current continuous approach. Nor have I exhausted the different models of discrete and discontinuous space and time that could be developed. However, once we countenance the rejection of continuity, a class of options open up with potential advantages for physical theory: the unification of a diversity of phenomena in the quantum domain, a novel approach to re-normalization problems, and so on. In brief, alternatives (1)-(4) do not exhaust the viable approaches to solving the *incipit/desinit* problem. The discreteness hypothesis at least deserves the further interest and serious investigation of physicists, mathematicians, and philosophers.

University of Adelaide

NOTES

1. *Cf.*, Frank Jackson and Robert Pargetter, "A Question About Rest and Motion," *Philosophical Studies*, Vol. 53, No. 1 (1988), pp. 141-46; Chris Mortensen, "The Limits of Change," *Australasian Journal of Philosophy*, Vol. 63, No. 1 (March, 1985), pp. 1-10; Graham Priest, *In Contradiction: A Study of the Transconsistent* (Dordrecht: Martinus Nijhoff, 1987); Joseph Wayne Smith, "Time, Change and Contradiction," *Australasian Journal of Philosophy*, Vol. 68, No. 2 (June, 1990), pp. 178-88.
2. Jackson and Pargetter, *op. cit.*, pp. 141-42.
3. This last alternative seems to capture Hegel's view of motion, though his pronouncements on the subject are obscure: "Something moves not because at one moment of time it is here and at another there, but because at one and the same moment it is here and not here" (A.V. Miller, *Hegel's Science of Logic* (London: George Allen and Unwin, 1969), p. 440)). For more on this Hegelian alternative *cf.*, Clark Butler, "Motion and Objective Contradictions," *American Philosophical Quarterly*, Vol. XVIII, No. 2 (April, 1981), pp. 131-39; George Melhuish, *The Paradoxical Nature of Reality* (Bristol: St. Vincent's Press, 1973), pp. 1-52; G.H. Von Wright, *Time, Change and Contradiction* (Cambridge: Cambridge University Press, 1968); Simo Knuuttila and Anja Inkeri Lehtinen, "Change and Contradiction: A Fourteenth-Century Controversy," *Synthese* Vol. 40 (1979), pp. 189-207.
4. Rejecting the infinite divisibility of space and/or time was common to the British empiricists: John Locke, *An Essay Concerning Human Understanding*, ed. A.S. Pringle-Pattison (Oxford: The Clarendon Press, 1964), Bk II, Chap. XV, Sec. 10; George Berkeley, *Berkeley's Philosophical Writings*, ed. David M. Armstrong (New York: Macmillan Publishing Co., 1965), pp. 116-117; David Hume, *Enquiries*, ed. L.A. Selby-Bigge (Oxford: The Clarendon Press, 1902), Bk. I, Pt. II, Sec 1. *Cf.* also, William James, *Some Problems of Philosophy* (Cambridge, Massachusetts: Harvard University Press, 1979), pp. 87-88; A.N. Whitehead, *Science and the Modern World* (Middlesex: Penguin Books, 1926), p. 105-106; Paul Weiss, *Reality*: (Princeton, N.J.: Princeton University Press, 1938), pp. 237-240.
5. Robert Blanche, *Contemporary Science and Rationalism* (Edinburgh:

Oliver and Boyd, 1968), pp. 33.

6. C.L. Hamblin, "Starting and Stopping," *The Monist*, Vol. LIII (1969), p. 412.
7. Aristotle, *Physica*, Trans. R.P. Hardie and R.K. Gaye (Oxford: The Clarendon Press, 1930), VI, 1, 231a.26.
8. Hamblin, *op. cit.*, pp. 410-25; C.L. Hamblin, "Instants and Intervals," in *The Study of Time*, ed. J.T. Fraser, F.C. Haber, and G.H. Muller (Berlin: Springer-Verlag, 1972), pp. 324-31.
9. *Cf.* also, L. Humberstone, "Interval Semantics for Tense Logics," *Journal of Philosophical Logic*, Vol. 8 (1979), pp. 171-96.
10. Cf., D. Hilbert and S. Cohn-Vossen, *Geometry and the Imagination*, Trans. P. Nemenyi (New York: Chelsea Publishing Co., 1952), pp. 33-34.
11. Jean Paul Van Bendegem, "Zeno's Paradoxes and the Tile Argument," *Philosophy of Science*, Vol. 54, No. 2 (1987), pp. 295-302.
12. W. Yourgrau, "Some Problems Concerning Fundamental Constants in Physics," in *Current Issues in the Philosophy of Science*, ed. Herbert Feigl and Grover Maxwell (New York: Holt, Rinehart and Winston, 1961), p. 327.
13. B. Abramenko, "On Dimensionality and Continuity of Physical Space and Time," *The British Journal for the Philosophy of Science*, Vol. IX, No. 34 (August, 1958), pp. 106-7.
14. It is possible to construct other units of length from different combinations of the constants c (the velocity of light *in vacuo*), h (Planck's constant of action), m (the rest-mass of the electron), e (the charge of the electron).

    For example:
    $h^3c/2\ {}^2me^4$: 914.10^{-5} cm. (Rydberg wave-length)
    $h^2/4\ {}^2me^2$: 529.10^{-8} cm. (first Bohr orbit of Hydrogen atom)
    h/mc : $2{\cdot}43.10^{-10}$ cm. (Compton wave-length for electron)
    e^2/mc^2 : $2{\cdot}82.10^{-13}$ cm. (electron radius).

    Any of these might serve as a suitable constant, given the appropriate theoretical setting. (*Cf.*, L.L. Whyte, "Fundamental Physical Theory: An Interpretation of the Present Position of the Theory of Particles," *The British Journal for the Philosophy of Science*, Vol. 1, No. 4 (February, 1951), pp. 303-27; G.J. Whitrow, *The Natural Philosophy of Time*, Second Edition (Oxford: The Clarendon Press, 1980), p. 204.)

15. *Cf.* R. Penrose and C.J. Isham (eds.), *Quantum Concepts in Space and Time* (Oxford: The Clarendon Press, 1986).
16. Emil J. Hellund and Katsumi Tanaka, "Quantized Space-Time," *Physical Review*, Vol. 94, No. 1 (April, 1954), p. 192.
17. C.L. Hamblin, "Starting and Stopping," p. 414.
18. C.J. Isham, R. Penrose, and D.W. Sciama, *Quantum Gravity 2* (Oxford: Oxford University Press, 1981), p. vii.
19. Priest, *op. cit.*, p. 204.
20. Von Wright, *op. cit.*, p. 22.
21. *Ibid.*, p. 30.
22. Graham Priest, "Inconsistencies in Motion," *American Philosophical Quarterly*, Vol. 22, No. 4 (October, 1985), p. 340.
23. *Ibid.*, p. 344.
24. *Ibid.*, p. 341.
25. Graham Priest, *In Contradiction: A Study of the Transconsistent*, p. 217. *Cf.* also, Henri Bergson, *Creative Evolution*, Trans. Arthur Mitchell (New York: Holt, Rinehart and Winston, 1911), p. 308: "every attempt to reconstitute change out of states implies the absurd proposition, that movement is made of immobilities."
26. Mortensen, *op. cit.*, pp. 2-3; Priest, *op. cit.*, p. 203.
27. *Cf.*, H. Margenau, *The Nature of Physical Reality* (New York: McGraw-Hill, 1950), p. 155.
28. *Cf.*, Hartland S. Snyder, "Quantized Space-Time," *Physical Review*, Vol. 71, No. 1 (January, 1947), pp. 38-41; Hartland S. Snyder, "The Electromagnetic Field in Quantized Space-Time," *Physical Review*, Vol. 72, No. 1 (July, 1947), pp. 68-71; Hellund and Tanaka, *op. cit.*, pp. 192-95. *Cf.* also Finkelstein's quantum-algebraic theory of spacetime: David Finkelstein, "Space-Time Code," *Physical Review*, Vol. 184, No. 5 (August, 1969), pp. 1261-1271; David Finkelstein, "First Flash and Second Vacuum," *International Journal of Theoretical Physics*, Vol. 28, No. 9 (1989), pp. 1081-1098.
29. Bertrand Russell, *Our Knowledge of the External World* (London: George Allen and Unwin Ltd., 1969), p. 140. *Cf.* also, G.E.L. Owen, "Zeno and the Mathematicians," in *Zeno's Paradoxes*, ed. Wesley C. Salmon (Indianapolis: Bobbs-Merrill Inc., 1970), p. 150; Reginald A.P. Rogers, "On Transfinite Numbers, and Some Problems Relating to the Structure of Actual Space and Time," *Her-*

mathena, Vol. XV (1909), p. 409.

30. Ilse Rosenthal-Schneider, *Reality and Scientific Truth: Discussions with Einstein, von Laue, and Planck* (Detroit: Wayne State University Press, 1980), p. 82.
31. *Cf.*, Erhardt W.R. Papp, "Quantum Theory of the Natural Space-Time Units," in *The Uncertainty Principle and Foundations of Quantum Mechanics: A Fifty Years Survey*, ed. William C. Price and Seymour S. Chissick (London: John Wiley and Sons, 1977), pp. 31-32.