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ANTI-REALISM AND OBJECTIVITY IN WITTGENSTEIN'S PHILOSOPHY OF MATHEMATICS

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Work on Wittgenstein's philosophy of mathematics is hampered by two problems. There is an interpretive problem of constructing a consistent account from Wittgenstein's various and diverse remarks and a philosophical problem of defending the radical nature of Wittgenstein's claims. I will offer an answer to a small part of these two problems. My particular interest is what is known as the "majority argument" against Wittgenstein's account of correct mathematical activity. I believe that this argument fails. However, there is a larger issue at stake. This argument is an instance of a presumed incompatibility of objectivity and anti-realism. It is often claimed that some version of realism is necessary for mathematical objectivity or, equivalently, that anti-realists cannot account for the objectivity of mathematics. Any full discussion of this claim must acknowledge that there are different notions of objectivity and, therefore, different versions of this objection[1]. I will discuss only one notion of objectivity which is said to rest on 'the distinction between appearance and reality,'[2] and is appealed to in the majority argument. I will conclude that this type of objectivity can be accounted for by anti-realists and in particular by Wittgenstein's philosophy of mathematics.

In the first section, I characterize realism and illustrate the sense in which Wittgenstein's account of mathematics is anti-realist. In the second section, I spell out the above notion of objectivity and show how an anti-realist account of truth, namely, Putnam's idealized rational acceptability, preserves objectivity. In the third section, I discuss the "majority argument" and illustrate how Wittgenstein's anti-realism can also account for the objectivity of mathematics. What Putnam's and Wittgenstein's anti-realisms ultimately show is that this notion of objectivity is distinct from the notion of realism and that an account of objectivity is no reason to be either realist or anti-realist.

I

Realism (with respect to a given theory or discourse) can be generally characterized as claiming that what makes the sentences of that theory or discourse true or false exists independently of us, our mental lives, our institutions, conventions and activities.[3] On this basis, Platonists are realist and intuitionists are anti-realist. Even though both typically claim that mathematical statements describe mathematical objects, the Platonist's objects are mind-independent while the idealist's are inner objects of thought. Empiricist accounts of mathematics also agree on regarding mathematical propositions as descriptive of mathematical reality. Let us generally refer to these accounts as "descriptivist" and allow them to differ on the nature of what is described. As the above example of idealism shows, descriptivism does not imply realism. However, realism (when it concerns the truth of sentences) plausibly implies descriptivism, if the following proviso is accepted: what makes a sentence true is described by that sentence.

I have raised the issue of descriptivism because Wittgenstein's criticisms of it offer key insights into his conception of mathematics. A realist account of mathematical truth needs to account for mathematical knowledge and the latter is closely connected with mathematical activity. Knowing mathematics and carrying on correct mathematical activities are closely linked. A major motivation for descriptivism lies in the apparently straightforward explanation it can offer for what makes certain mathematical activities correct in terms of our knowledge of relevant mathematical truths or of the agreement between these activities and truths. It is this descriptivist explanatory hypothesis which Wittgenstein regards as both insufficient and unnecessary:

> When we ask what inferring consists in, we hear it said e.g.: "If I have recognized the truth of the propositions..., then I am justified in further writing down..." — In what sense justified? Had I no right to write that down before? — "Those propositions convince me of the truth of this proposition." But of course that is not what is in

> question either. — "The mind carries out the special activity of logical inference according to these laws." That is certainly interesting and important, but then, is it true? Does the mind always infer according to *these* laws? And what does the special activity of inferring consist in? — This is why it is necessary to look and see how we carry out inferences in the practice of language; what kind of procedure in the language-game inferring is. (RFM I, 17)[4]

What Wittgenstein says here about inferring applies as well to activities like counting or calculating. Wittgenstein questions how any propositional knowledge can explain why an action is correct. For example, how can knowing *that* two plus two equals four explain the correctness of saying "four" after counting successively two disjoint pairs of objects. This is the general question that Wittgenstein raises against descriptivist accounts of mathematics. Take our explanandum as the correctness of some mathematical activity. Descriptivists claim the explanans is the knowledge of the relevant mathematical truth. Wittgenstein claims that the latter can only convince us of certain other mathematical truths and 'that is not what is in question.' Exactly how one's knowledge claims can explain the correctness of one's actions is left unclear. Perhaps, it might be said that descriptivism alone is an insufficient explanation of our mathematical activity, for it requires supplement by something like a causal theory of action where relevant states of knowledge can cause actions. However, the problem can be located at an even deeper level. Wittgenstein claims that the descriptivist explanans is not even necessary to account for any of our correct mathematical activities. Referring to our knowledge of mathematics, Wittgenstein asks 'does the mind always infer according to (or because of) these laws?' The implicit answer is no. We may learn to do mathematics without knowing any mathematical statements. We do not literally need to have any mathematical propositional knowledge, i.e., knowledge that such-and-such mathematical proposition is true.

> If somebody calculates like this must he utter any "arithmetical *proposition*"? Of course, we teach children the multiplication tables in the form of little *sentences*, but is that essential? Why shouldn't they simply: *learn to calculate*? And when they can do so haven't they learnt arithmetic? (RFM I, 144)

Our correct performance of mathematical activities can be explained simply by our learning how to do mathematics. If this is so, then we do not need to learn any mathematical truths which describe a mathematical reality; rather, we need only learn a certain activity. As a result, descriptivism is irrelevant for explaining correct mathematical activity.

One of Wittgenstein's major claims is that the philosophy of mathematics must focus on an account of correct mathematical practice, and not on an account of mathematical truth. Consider the following two possibilities. Imagine a person, Abner, who has memorized all the true arithmetic statements in which any two integers which he can comprehend are added together. Abner can correctly assent to any true statement such as "7+8=15", and dissent to any identical addition with a different result such as "7+8=9". However, Abner cannot actually do arithmetic. When given an addition problem, Abner cannot calculate the addition of two integers because he has not learned the connection between statements of the above form and the activity of calculating. Abner cannot be said to know arithmetic. Compare Abner with Alice, who has learned to calculate the sum of any two integers and detect mistakes in any incorrect calculation. However, Alice cannot assent or dissent to any arithmetic statement in which two integers are added. Like Abner, Alice has not learned the connection between arithmetic statements and the activity of calculating. Despite this, for Wittgenstein, Alice knows arithmetic. Alice has learned that, e.g., 2+2=4, not because she has learned something about mathematical objects, but rather because she has learned to say "4" after counting successively two pairs of disjoints objects. Once this point is appreciated, the focus for a philosophy of mathematics changes radically. The importance of mathematical statements lies not in their describing what is true or false, bur rather in their prescribing what is correct or incorrect in our mathematical activities.[5]

If the central function for mathematical statements is to prescribe correct mathematical practice, then these statements are best regarded as rule-like or normative. Since rules cannot be true or false, it is not fully accurate to say that Wittgenstein accounts for the *truth* of mathematical statements. Instead, he accounts for the *correctness* of mathematical rules. This account is trivially anti-realist: mathematical statements are not literally true or false; hence, there exists no reality which makes mathematical statements true or false. However, one can deny that mathematical statements are true or false, and yet claim that the correctness of mathematical rules depends on some mathematical reality independent of us; thereby, becoming a realist with respect to correctness.[6] Concerning the correctness of rules, realism (with respect to a given theory or discourse) can be characterized as claiming that what makes the rules of that theory or discourse correct or incorrect exists independently of us, our mental lives, our institutions, conventions, and activities. This allows mathematics to be normative (not descriptive) and yet its correctness depends on some reality independent of us. In this manner, a non-descriptivist can be a realist.

Because of his focus on mathematical practice, Wittgenstein rejects descriptivism and regards mathematical statements as rules. Furthermore, since he accounts for the correctness of these rules solely in terms of our conventions and behavior, he rejects realism:

> ...what we call "counting" is an important part of our life's activities. Counting and calculating are not — e.g. — simply a pastime. Counting (and that means: counting like *this*) is a technique that is employed daily in the most various operations of our lives... — "But is this counting only a *use*, then; isn't there also some truth corresponding to this sequence?" The *truth* is that counting has proved to pay. — "Then do you want to say that 'being true' means: being usable (or useful)?" — No, not that; but that it can't be said of the series of natural numbers — any more than of our language — that it is true, but: that it is usable, and, above all, *it is used*. (RFM I, 4)

> And of course there is such a thing as right and wrong in passing from one measure to the other; but what is the reality that 'right' accords with here? Presumably a *convention*, or a *use*, and perhaps our practical requirements. (RFM I, 9)

On one level, Wittgenstein accounts for the correctness of mathematical activities, e.g., "counting like *this*," in terms of our actual practice. In other words, from a range of possible mathematical activities, the correct ones are indicated by what we actually use. But this is not to say that we could have chosen any practice as the one for us. On a deeper level, Wittgenstein accounts for the correctness of mathematical activities in terms of what has "proved to pay" and our "practical requirements." In this sense the mathematical activities that we actually use are not arbitrarily chosen.

This deeper Wittgensteinian account of correct mathematical activity rests on his notion of a form of life. With respect to mathematics, I interpret this to involve three factors — namely, (i) the role of mathematics in our lives, (ii) our physical and mental features, and (iii) the natural features of the world. The joint influence of all these factors, and not some mathematical reality independent of us, determines what is and what is not correct counting, calculating, and measuring. Think of the composition of factors (i) through (iii) as analogous to the composition of forces in mechanics. These are governed by the rule of the parallelogram. A modification in the intensity or direction of any side is going to affect the resulting diagonal force, but no one side is sufficient to determine the resulting force. Analogously, neither (i) nor (ii) nor (iii) alone is sufficient to determine correct mathematical activity. This allows us to see how something independent of us, e.g., a change in the natural features of the world, can affect our mathematical activity. Suppose that the behavior of physical objects changes so that all aggregates of physical objects increased by one unit at temperatures warmer than 50 Fahrenheit. Two apples plus three apples equals five apples when the temperature is less than 50 degrees and six apples when the temperature is warmer. Our mathematical activity would change only if the role of mathematics in our lives and our physical and mental features stayed the same. Natural features of the world alone cannot determine correct mathematical activity. The criterion of correctness for mathematical activities is the subtle and interesting interaction of all three factors I list above. They comprise what I take to be the key elements of Wittgenstein's "form of life."[7] In this sense, the correctness of mathematical activities depends on us.

In this section, I discussed the notion of realism and its relation with descriptivism. Wittgenstein's rejection of descriptivism as well as his focus on mathematical activity led us to an alternative version of realism. In conclusion, I have outlined Wittgenstein's account of the correctness of mathematical activities and offered a further interpretation to make fully explicit how this account is anti-realist.

II

Objectivity, on one well-established use of the term, is located in the

> distinction between appearance and reality; to maintain that it is an objective matter whether or not a certain speaker's claim is true is, on this use, to maintain that there is a clear difference between the claim merely seeming to be true to the speaker, and its actually being true.[8]

In this passage, objectivity lies in the distinction between a matter's seeming to be true to somebody, or even to a whole society, and its actually being true. Accordingly, I spell out this notion of objectivity as claiming that for any statement S, the conditions for S's truth are different from the conditions for S's seeming to be true. This distinguishes between appearance and reality in terms of the conditions of truth for statements and the conditions of their seeming to be true. Although there may be other interesting notions of objectivity, I only discuss this one because it is often appealed to in the critical literature on Wittgenstein's account of mathematics.

One straightforward way to maintain a difference between a claim's seeming to be true and its actually being true is to argue that there is something independent of us on which the truth of the claim depends. If a statement is true, it may not seem to be true on the basis of our procedures. Also, if a statement seems to be true on the basis of our procedures, it may be false. Realism thereby quite easily distinguishes between the conditions for a claim's truth and its seeming to be true by making them logically independent.[9]

The fact that realism provides a very straightforward account of this distinction might suggest that this notion of objectivity requires realism. However, objectivity and realism are distinct. It is possible to distinguish the conditions for a claim's truth from the conditions for a claim's seeming to be true without implying that the conditions of a claim's truth depend on some reality independent of us. At least two accounts of truth can avoid this identification without any commitment to realism: the coherence theory of truth and Hilary Putnam's theory of truth as an idealization of rational acceptability.[10] In the remainder of this section, I will discuss only Putnam's account. We shall see that his account provides another straightforward distinction between conditions of truth and conditions of seeming truth. This will introduce the discussion of the next section, which shows how Wittgenstein's antirealist account of the criteria of correctness for mathematical activity can also account for this notion of objectivity.

When our procedures for determining the truth-value of statements lead us to regard a certain statement as "rationally acceptable", this means that as far as our system for checking and verifying our beliefs is concerned, we are justified in holding such a statement as true. Now, quite clearly, this notion of rational acceptability is not the same notion we have in mind when we say that a certain claim is true. For, rational acceptability, being grounded on justification, is a matter of degree, is relative to persons and is tensed. The belief in a certain claim may be highly justified for somebody at a certain time, but may be poorly or not at all justified for the same (or some other) person at a different time. But truth is not relative to persons or to times, nor is it a matter of degrees.[11] Truth cannot be fully accounted for in terms of the conditions under which we are justified in holding a belief as true. Our being, however highly, justified in believing something is not sufficient for truth. Hence, a claim's being true and its seeming to be true are distinct.

Putnam maintains the distinction between seeming truth and truth required to explain objectivity. According to Putnam, truth is an idealization of rational acceptability, i.e., it is what beings like us would hold as true, were they under epistemically ideal conditions. Truth is what an ideal knower would be justified in believing were the epistemically ideal conditions for justification to hold. This account of truth is just as effective as the above realist account in explaining the logical independence of truth from our actual judgments. It may be the case that we regard as rationally acceptable a false claim, i.e., a claim that we would not regard as rationally acceptable under ideal epistemic conditions. Conversely, it may as well be that we do not regard as rationally acceptable a true claim, i.e., a claim that we would recognize as rationally acceptable under the ideal epistemic conditions. In Putnam's account, truth is independent of our actual judgments, but it is not independent of the notion of judgment itself. As he says, "truth is independent of justification *here* and *now*, but not independent of *all* justification". Truth is equivalent to "idealized" justification.[12] This distinction between truth and our actual justification maintains objectivity.

In conclusion, Putnam presents his account of truth as an idealization of rational acceptability as an alternative to the "correspondence" theory of truth of metaphysical realism. It is, of course, an interesting question whether his alternative is plausible. However, this issue is irrelevant to the issue of objectivity. To claim that realism is necessary for this notion of objectivity because it is necessary for a correct account of truth simply collapses the objectivity objection into a discussion of an account of truth and begs the question against the anti-realist. I will expand upon this point in the conclusion of this paper.

III

There is an argument which occurs often in the critical literature on Wittgenstein.[13] It concerns rule-following behavior and claims that Wittgenstein's account of correct rule-following behavior, e.g., mathematical activity, cannot preserve objectivity. It is sometimes referred to as the "majority" argument because it focuses on the judgements performed by all or the majority of the members of a society. The following is my reformulation:

(MA)

1. If Wittgenstein's account of following rules is correct, then the criteria of correctness for mathematical activities depend on social practices. E.g., the next correct number for continuing the sequence, '2, 4, 6, ...' is determined by the practice that all or the majority of the members of the society follow in continuing such a sequence.

2. If the consequent of 1. is true, then for any application S in following the mathematical rule R, S is correct if and only if S seems to be correct to all or the majority of the members of the society.

3. For any application S in following the mathematical rule R, S is objectively correct if and only if it is not the case that S is correct if and only if S seems to be correct to all or the majority of the members of the society.

4. ... If Wittgenstein's account of following rules is correct, then the criteria of correctness for mathematical activities are not objective.

Premise three reformulates the notion of objectivity with regard to the correctness of the applications of mathematical rules, i.e., mathematical activity. This notion distinguishes appearance (what seems to be correct) from reality (what is correct). This argument is very persuasive. As we have seen, for Wittgenstein, questions about the correctness of mathematical activity are answered by its use. In other words, if we carry on a particular mathematical activity, it is correct. Generally speaking, if something seems to us to be a correct mathematical activity, we do it; hence, Wittgenstein's criteria of correctness for mathematical activity reduce to what seems to be correct to the majority and lose objectivity. In the remainder of this paper, I offer a Wittgensteinian account of correct mathematical rules which distinguishes what seems to be correct from what is correct, thereby, preserving objectivity.

As I have argued in section II, Wittgenstein allows various factors to determine the correct mathematical procedures. First, there is the role that mathematics has in our life and its fundamental usefulness for our survival and development. Second, there are the physical and mental features that makes us the beings we are, i.e., beings that can communicate in certain way, think as we do, with certain needs, interests and wants. Third, there are the features of our world: its physical composition and regularities. These influences have shaped and developed mathematical activity as we know it today. Not any way of counting and calculating can do equally well for the needs of beings with our physical and psychological structure, who evolve in a world like ours. It is the particular way we count or calculate which works for us.[14]

In one sense, it is true that, for Wittgenstein, the criterion of correctness for mathematical activity is simply the fact that this is the way we do mathematics. However, Wittgenstein allows for an account of the way we do mathematics in broad evolutionary terms (understood as appealing to the above three factors). If the criteria of correctness can be understood in terms of this deeper account, i.e., in connection with the goal of the survival and development of human life on the earth, then these criteria are evolutionary factors which need not coincide with our actual practices. Under conditions of changing evolutionary pressures, it may be the case that the mathematical practice we actually follow does not work for us.

Let us make this point clearer by means of an extreme example. Consider the case of a nuclear explosion affecting our whole planet. Suppose that, totally unbeknown to the survivors and in consequence of the explosion, their brain structure has radically changed. Although they believe to be correctly performing mathematical calculations and although they experience all the psychological states that they remember accompanied their calculating activities before the explosion, actually they are merely listing numbers at random. As a result, the mathematical practice that seems to be correct to the majority or all of the survivors is not the practice which is actually correct. This would make itself startingly apparent if they used mathematical procedures for practical purposes, such as building bridges and houses, and preparing medicines, etc. Their bridges and houses would collapse and their medicines would poison them. The criterion of correctness for mathematical activity is simply that which works for us. By hypothesis, in the nuclear explosion example, actual practice, or what seems to be correct, does not work, i.e., does not coincide with what is correct. Eventually, mathematical practice must undergo a change, or be abandoned, or the survivors would perish. The contrast between actual practice (what seems to be correct to all or most of us) and what works for us (what is correct) is all that Wittgenstein needs to allow for the objectivity of the criteria of correctness of mathematical rules.

The majority argument fails against Wittgenstein. As its root is the conviction that if the criterion of correctness for a practice is the result of some social procedure of decision, then the distinction between appearance and reality is lost. After all, social procedures merely record what seems to be correct to the majority of society and this needn't necessarily be what is correct. However, a socially agreed upon practice can embody criteria of correctness different from the mere approval of the majority or totality of society such as an evolutionary demand that these practices work.

In conclusion, anti-realist accounts of truth, like Putnam's and anti-realist accounts of correct mathematical activities, like Wittgenstein's can account for a notion of objectivity which distinguishes between appearance and reality. This result diffuses a popular argument against Wittgenstein. However, I have only shown that a theory can distinguish between what seems to be true, or correct, and what is true, or correct, and construe the latter in anti-realist terms. Indeed, this should be easy for any non-subjectivist theory. This leaves open the question of the best account of what is true, or correct. Many philosophers take some version of realism to be the best answer, although I disagree. The positive result of this paper is that accounting for the above notion of objectivity is no reason for agreeing with them.[15]

NOTES

1. In "The Argument form Agreement and Mathematical Realism," I discuss a version of this objection which appeals to an epistemic notion of objectivity, defined as agreement on results.
2. S.H. Holtzman and C.M. Leich, eds., *Wittgenstein: To Follow a Rule* (London: Routledge, 1981) p. 2.
3. This definition is drawn form two sources: H. Putnam, "What is Mathematical Truth?", in *Philosophical Papers*, Vol. I (Cambridge: Cambridge University Press, 1975): 60-78, 69-70 and M.D. Resnik, *Frege and the Foundations of Mathematics* (Ithaca: Cornell University Press, 1980) p. 162. In the contemporary debate, bivalence and verification-transcendence often appear in definitions of realist accounts of truth. The first requires that every statement of a theory be true or false; the second requires that statements have a truth-value independently of our ability to know or verify it. In my characterization of realism, I use independence in a stronger sense which requires that the truth-value of the statements of a theory be independent not only of our cognitive life, but also of our institutions, conventions and activities. A conventionalist account which denies that we *know* any mathematical truths satisfies the condition of verification-transcendence and would be thus classified as realist, although our conventions and practices and not any mathematical reality determine the truth-value of mathematical statements. Such an account is more plausibly classified as anti-realist under my characterization of realism. Finally, I do not regard bivalence as a necessary condition for realism. On this point, Alvin Goldman's brief discussion of bivalence is quite convincing, see *Epistemology and Cognition* (Cambridge: Harvard University Press, 1986) p. 143.
4. L. Wittgenstein, *Remarks on the Foundations of Mathematics*, ed. G.H. von Wright, R. Rhees, G.E.M. Anscombe (Cambridge: The MIT Press, 1983). Quotations from this work are followed by RFM, a Roman numeral indicating the part of the book and an Arabic numeral indicating the numbered remark.

5. I owe this helpful example to Lory Lemke.
6. This shows that truth and realism are independent notions. This point is clearly stated and convincingly defended by the supporter of a form of wholehearted realism, i.e., Michael Devitt. See his "Dummett's Anti-Realism", *Journal of Philosophy*, 80 (1983), 73-99, section I,1; "Realism and the Renegade Putnam," *Nous*, 17 (1983), 291-301, section 1; and *Realism and Truth* (Princeton, New Jersey: Princeton University Press: 1984) chapter 4.
7. My interpretation is closer to Barry Stroud's than to Michael Dummett's reading of the *Remarks*. Stroud defends this interpretation and discusses Dummett's in "Wittgenstein and Logical Necessity" in G. Pitcher, ed., *Wittgenstein. The Philosophical Investigations* (Notre Dame: University of Notre Dame Press, 1968): 477-96. Dummett states his view in "Wittgenstein's Philosophy of Mathematics", also in G. Pitcher, *op.cit.*, 425-49
8. S.H. Holtzman and C.M. Leich, eds., *Wittgenstein: To Follow a Rule*, *op.cit.*, *ibidem*.
9. '... to repeat, the root idea of objectivity is that truth is not constituted by but is somehow independent of human judgment. Realism gives this independence the obvious interpretation: *logical* independence — the idea that for particular true statements it is either unnecessary or insufficient, or both, to meet our most refined criteria of acceptability in order to be true.' C. Wright, *Wittgenstein on the Foundations of Mathematics* (Cambridge, Mass.: Harvard U.P., 1980) p. 199.
10. N. Rescher, *The Coherence Theory of Truth* (Oxford: Clarendon Press, 1973); H. Putnam, *Reason, Truth and History* (Cambrdige: Cambridge U.P., 1981) Chapter 3.
11. 'Truth cannot simply *be* rational acceptability for one fundamental reason; truth is a property of a statement that cannot be lost, whereas justification can be lost. The statement "The earth is flat" was, very likely, rationally acceptable 3,000 years ago; but it is not rationally acceptable today. Yet it would be wrong to say that "the earth is flat" was *true* 3,000 years ago, for that would mean that the earth has changed its shape. In fact rational acceptability is a matter of degree; truth is sometimes spoken of as a matter of degree (e.g., we sometimes say *"the rearth is a sphere" is approximately true* but the "degree" here is the *accuracy* of the statement, and not its degree of

acceptability or justification).' H. Putnam, *Reason, Truth, and History*, *op.cit.*, p. 55.

12. In Rescher's coherence theory of truth, "real truth" as an idealization of the "truth we believe here and now" plays a role very close to the one that ideal rational acceptability plays in Putnam's view, see *The Coherence Theory of Truth*, *op. cit.*, pp. 181-185.
13. For a particular statement of this argument (which is discussed elsewhere in the same works), see Wright, *op. cit.*, 217-22; Holtzman and Leich, *op. cit.*, p. 3; J. McDowell, "Wittgenstein on Following a Rule", *Synthese* 58 (1984), 325-63, 328.
14. See RFM I, 4 and 9, quoted in Section I.
15. Earlier drafts of this paper were read at the 1985 meeting of the Minnesota Philosophical Society, and at the 1986 APA Central Division Meeting. I thank both my commentators, Sandra Peterson and John Koethe, respectively, for stimulating criticisms and comments. I am especially grateful to Lory Lemke for his careful criticism of previous drafts. Thanks also to Ute St. Clair and Ted Uehling for helpful discussions.