INTRODUCTION

This is the second volume on *Recent Issues in the Philosophy of Mathematics*. As the reader will notice, perhaps *Recent Issues in the Philosophy and History of Mathematics* would have been a better choice. The disadvantage of that option is that one might get the impression that the contributions brought together belong to two different fields and that hardly any interrelations exist. But that would surely be a false picture. The historical contributions in these volumes explicitly deal with philosophical issues and the philosophical contributions rely on historical evidence to support the theses presented. But, as an editor, I cannot resist an attempt at classification however inadequate and distorting. In the best of cases, it may help the reader to detect the new developments in the philosophy-history-sociology (or any permutation thereof) of mathematics.

The more historical contributions show, I dare say quite clearly, that the 'standard' version is not the whole truth of the matter. Sometimes, as the standard version wants us to believe, mathematics does seem to grow in a linear, cumulative fashion but one should add 'locally' and 'for a limited period of time' as qualifications. Both Jens Høyrup (*Jordanus de Nemore: A Case Study on 13th Century Mathematical Innovation and Failure in Cultural Context*) and Irving H. Anellis (*Distortions and Discontinuities of Mathematical Progress: A Matter of Style, A Matter of Luck, A Matter of Time, ... A Matter of Fact*) present us with case studies that show that both accidental elements - as in Jean Van Heijenoort's case - and being ahead of one's time - the reason why the work of Jordanus de Nemore failed to have the impact it should have had - can cause deviations in the historical course of mathematics. In Eduard Glas' contribution (*Between Form and Function. Social Issues in Mathematical Change*), it is fascinating to see how political and sociological influences are clearly related to mathematical style, how mathematics is done, and how it is taught. His case study of revolutionary France is a convincing piece of historical work. Erkka Maula and Eero Kasanen (*Chez Fermat A.D. 1637*) proceed in an entirely different way. Perhaps the best way to illustrate the difference with the other historical contributions, is to make a comparison with music. What is better? To perform Bach on present-day instruments, arguing that since our instruments are better and technically substantially improved, the musical result must therefore be better. Or, to perform Bach on the original instruments, the argument being that only then can we appreciate the full musical meaning Bach intended and, who knows, discover an unknown interpretation. Maula and Kasanen perform such a suite on Fermat's Last Theorem using original instruments only. Whether or not one agrees with this kind of approach, it is a quite interesting and stimulating attempt combining historical insights and mathematical technique. Apart from this difference, this editor for one was quite amazed to see that all the authors mentioned so far, tend to agree that we should go beyond the rationalist reconstructionist mathematical philosophy of Imre Lakatos. I am very willing to share this view - if history is cruel and mean, then we should say so - but I do hope that the importance of *Proofs and Refutations* will not be forgotten.

In the more philosophical contributions, I discern at least three major approaches: (a) What has modern philosophy of science to tell us about mathematics? Are models and theories designed to deal with problems of scientific theories also suited for dealing with mathematical theories? (b) What is the impact of the computer (metaphor) on mathematical practice and on our representation of the process of thinking? and (c) What relevance do results in formal philosophy of mathematics and logic have for particular philosophical issues? And, more generally, how does the philosophy of mathematics relate to mathematics itself?

Both papers by Yehuda Rav (*Philosophical Problems of Mathematics in the Light of Evolutionary Epistemology*), and Michael D. Resnik (*A Naturalized Epistemology for a Platonist Mathematical Ontology*), look at the developments in evolutionary and/or naturalized epistemology, and investigate what we can learn from it in the mathematical context. Both find many quite interesting links and offer a new way to look at the development of mathematics. However, both also make absolutely clear that mathematics does have its own special status. Resnik is a clear case at hand: he develops his views on naturalized mathematical epistemology with the purpose of defending his version of mathematical platonism.

In this respect, Charles F. Kielkopf (*Fallible Intuitions: The Apriori in Your Mathematics*) presents a similar attempt. On the one hand, he agrees with Philip Kitcher that the mathematical a priori does not exist, but, on the other hand, he is clearly not satisfied with this state of affairs. His way out is to make a distinction between singular and communal mathematical knowledge, arguing that Kitcher's analysis applies only to the communal level, and that the mathematical a priori does make sense on the singular, personal level.

Lest one should think that all modern philosophers of mathematics hold similar views, Sal Restivo's contribution (*The Social Life of Mathematics*) confronts us with a deep-going sociological point of view. 'Rationality and well-founded reasoning cannot be separated from social action and culture' (first volume, p.15) summarizes his position. The lecture of his article is best followed by the historical-sociological study of Eduard Glas, already mentioned. Together they form, in my mind, a very convincing case for, to use Restivo's term, the strong sociology of mathematics.

It is obvious that a lot of work still needs to be done here. Traditional philosophy of mathematics is undergoing some fundamental changes. It is rather unlikely that an essay on some philosophical problem about mathematics will not make reference to sociological, psychological and historical matters. This can only be interpreted as a new stage in the philosophy of mathematics.

A second new development is the impact of the computer and its use in mathematical practice. James Franklin (*Mathematics, The Computer Revolution and the Real World*) ends his article with a list of problems to solve or to think about. It is striking to see that most of these problems could not even have been formulated, say, thirty or forty years ago. The computer is radically changing our worldview and, as Franklin points out, even 'pure' mathematics does not remain immune. Stuart G. Shanker (*The Dawning of (Machine) Intelligence*) sketches a broader picture and presents a very intriguing analysis explaining the behavioral roots of the concept of the Turing machine and its subsequent influence in cognitive science.

The last group of papers makes reference, as indicated, to results in formal logic and formal foundations of mathematics and tries to apply or to interpret these results in the framework of a philosophical problem. Thus, Thomas Tymoczko's paper (*Mathematical Skepticism: Are We Brains in a Countable Vat?*) rests entirely on an interpretation of the Löwenheim-Skolem (downward) theorem within the context of the problem of skepticism. In a way, he continues a tradition that started with Lucas' interpretation of Gödel's theorem. In my own paper (*Foundations of Mathematics or Mathematical Practice: Is One Forced To Choose?*), I have tried to apply the notion of artificial mathematician, so common in the foundations of mathematics, to the real mathematician in order to better understand the differences between Mathematics (with capital m) and real mathematics.

Both J. Fang (*Between Philosophy and Mathematics: Their Parallel on a "Parallax"*) and Michael Otte (*Der Charakter der Mathematik zwischen Philosophie und Wissenschaft*) draw a broader picture. Both stress the fact that the developments in the first half of this century - say, the foundational movement, lacking a better term - has seriously distorted the relation between mathematics and philosophy of mathematics. The latter was mainly interested in the formal foundations, thereby ignoring, as Fang shows, what was going on in daily mathematical life. Otte wants us to reflect on the strange nature of mathematics, on what he calls, its *Doppelcharakter* (its two-sided nature). Basically, as I understand it, when a finite mind thinks about an infinite set, according to your point of view, you are either dealing with a finite or with an infinite process. The point is not that we have to choose, the point is that mathematics by its very nature generates the two possibilities.

Perhaps the reader will remark that this set of papers is seriously incomplete: where is Ludwig Wittgenstein? True, he is not present explicitly, but there can be no question about his implicit presence. Many of the problems raised, many of the remarks made are almost literally derived from the *Bemerkungen*. And, finally, some solitary reader might remark that, apart from Wittgenstein, another important philosopher of mathematics should not be forgotten: Edmund Husserl. But then, though somewhat dimmer, he is present too.

Jean Paul Van Bendegem