THE SOCIAL LIFE OF MATHEMATICS

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I. *Introduction*

Mathematicians and philosophers of mathematics have long claimed exclusive jurisdiction over inquiries into the nature of mathematical knowledge. Their inquiries have been based on the following sorts of assumptions: that Platonic and Pythagorean conceptions of mathematics are valid, intelligible, and useful; that mathematics is a creation of pure thought; and that the secret of mathematical power lies in the formal relations among symbols. The language used to talk about the nature of mathematical knowledge has traditionally been the language of mathematics itself. When other languages (for example, philosophy and logic) have been used, they have been languages highly dependent on or derived from mathematics. By contrast, social talk takes priority over technical mathematical talk when we consider mathematics in sociological terms.

Sociological thinking about mathematics has developed inside and outside of the mathematical community. In the social science community, it is manifested in insider, professional sociology. In the mathematical community, the everyday folk sociology of mathematicians became better articulated as mathematical work became better organized and institutional continuity was established beginning in seventeenth century Europe. Eventually, some mathematicians who were especially self-conscious about the social life of the mathematical community began to write social and even sociological histories of their field (Dirk Struik, for example), or exhibit that self-consciousness in their mathematical programs (as in the cases of the constructivists, and the group of mathematicians known as N. Bourbaki). It is therefore no longer obvious that technical talk can provide a complete understanding of mathematics.

The decline of Platonic, Pythagorean, formalist, and foundationalist prejudices has opened the door for social talk about mathematics. But the implications and potential of social talk

about mathematics have yet to be realized. My task in this paper
is programmatic: it is to sketch the implications and potential of
thinking and talking about mathematics in sociological terms.

There is a sociological imperative surfacing across a wide
range of fields that is changing the way we view ourselves, our
world, and mathematical and scientific knowledge. This is not
disciplinary imperialism; the sociological imperative is not the
same as the discipline or profession of sociology. It is a way of
looking at the world that is developing in the context of the
modern experience, a mode of thought emerging out of modern
social practice. The basis for this Copernican social science
revolution lies in three interrelated insights: *all* talk is *social*;
the person is a *social structure*; and the intellect (mind, con-
sciousness, cognitive apparatus) is a *social structure*. These
insights are the foundation of a radical sociology of mathematics.

II. *Intellectual Origins of the Sociological Imperative.*

The program for a radical sociology of mathematics, in which all
talk about mathematics is social talk, begins with Marx's formula-
tion of the insight that science is a social activity. In order to
underline the theme of this paper, I take the liberty of substi-
tuting the term "mathematical" for "scientific" in the following
quotation:

> Even when I carry out [*mathematical*] work, etc., an ac-
> tivity which I can seldom conduct in direct association with
> other men - I perform a *social*, because *human*, act. It is
> not only the material of my activity - like the language
> itself which the thinker uses - which is given to me as a
> social product. My *own* existence *is* a social activity.[1]

Durkheim argues that individualized thoughts can only be
understood and explained by attaching them to the social condi-
tions they depend on. Thus, ideas become communicable concepts
only when and to the extent that they can be and are *shared*.[2]
The laws of thought and logic that George Boole searched for in
pure cognitive processes (see the discussion below) are in fact
to be found in social life. The apparently purest concepts,
logical concepts, take on the appearance of objective and imper-
sonal concepts only to the extent that and by virtue of the fact
that they are *communicable* and *communicated* - that is, only
insofar as they are *collective representations*. All concepts, then,
are collective representations and collective elaborations because

they are conceived, developed, sustained, and changed through social *work* in social contexts. In fact, *all* contexts of human thought and action are social. The next intellectual step is to recognize that "work", "context", "thought", and "action" are inseparable; concepts, then, are not *merely* social products, they are *constitutively* social. This line of thinking leads to the radical conclusion that it is social worlds or communities that think, not individuals. Communities as such do not *literally* think in some superorganic sense. Rather, individuals are *vehicles* for expressing the thoughts of communities or "thought collectives". Or, to put it another way, *minds are social structures*.[3]

It is hopeless to suppose that social talk insights could be arrived at or appreciated by people immersed in technical talk. In order to understand and appreciate such insights, one has to enter mathematics as a completely social world rather than a world of forms, signs, symbols, imagination, intuition, and reasoning. This first step awakens us to "math worlds", networks of human beings communicating in areas of conflict and cooperation, domination and subordination. Here we begin to experience mathematics as social practice, and to identify its connections to and interdependence with other social practices. Entering math worlds *ethnographically* reveals the continuity between the social networks of mathematics and the social networks of society as a whole. And it reveals the analogy between cultural production in mathematics and cultural production in all other social activities.[4]

The second step in this sequence occurs when we recognize that social talk and technical talk seem to be going on simultaneously and interchangeably. The third step brings technical talk into focus in terms of the natural history, ethnography, and social history of signs, symbols, vehicles of meaning, and imagination. The more we participate in the math worlds in which mathematicians "look, name, listen, and make", the more we find ourselves despiritualizing technical talk.[5] The final step in comprehending the sociological imperative occurs when we realize at last and at least in principle that technical talk *is* social talk.

Mathematical knowledge is not simply a "parade of syntactic variations", a set of "structural transformations", or "concatenations of pure form". The more we immerse ourselves ethnographically in math worlds, the more we are impressed by the universality of the sociological imperative. Mathematical forms or objects increasingly come to be seen as sensibilities, collective formations, and worldviews. The foundations of mathematics are not located in logic or systems of axioms but rather in social life. Mathematical forms or objects *embody* math worlds. They contain

the social history of their construction. They are produced *in* and *by* math worlds. It is, in the end, math worlds, not individual mathematicians, that manufacture mathematics.[6]

Our liberation from transcendental, supernatural, and idealist visions and forces begins when the sociological imperative captures religion from the theologists and believers and unmasks it; it becomes final (for this stage of human history) when that same imperative takes mind and intellect out of the hands of the philosophers and psychologists. It is in this context of inquiry that the larger agenda of the sociology of mathematics becomes apparent. Durkheim set this agenda when he linked his sociological study of religion to a program for the sociological study of logical concepts. The first full expression of this agenda occurs in Oswald Spengler's *The Decline of the West.*

III. *Spengler on Numbers and Culture*

In one of the earliest announcements of a "new sociology of science", David Bloor mentions Oswald Spengler as one of the few writers who challenges the self-evident "fact" that mathematics is universal and invariant.[7] But Bloor says little more about Spengler's chapter on "Numbers and Culture" in *The Decline of the West* than that it is "lengthy and fascinating, if sometimes obscure". Length and obscurity are apparently two of the reasons Spengler has been ignored as a seminal contributor to our understanding of mathematics. He is also considered too conservative, and even a fascist sympathizer and ideologue, by some intellectuals and scholars and thus unworthy of serious consideration as a thinker. But Spengler was not a fascist, and certainly no more conservative or nationalistic than other scholars and intellectuals who have earned widespread respect in the research community (Max Weber, for example). And the interesting affinities between Spengler and Wittgenstein, and in fact Spengler's influence on this central figure in the pantheon of modern philosophy, are only now beginning to come to light. Of special interest in this respect is their common, and widely overlooked, ethical agenda.[8] But it is their common vision of an anthropology of mathematics that is of immediate interest.[9]

There can be little doubt that part of the reason for the resistance to Spengler is that unlike other writers who have challenged the central values of Western culture (including Wittgenstein), Spengler is harder to address as a reasonable and recognizable opponent or ally. Contradictions and paradoxes in his work aside, he does not really want to play the games of

modern science, culture, and philosophy. Those who do want to play these games cannot really "use" Spengler, even when they somehow can appreciate him. Bloor is a case in point. Whatever his admiration for and indebtedness to Spengler, in the end Bloor's sociology of mathematics is grounded in a defense of modern (Western) science and modern (Western) culture.[10]

Spengler's analysis does not assign a privileged status to Western culture or Western science. It is also important to recognize the significance of the priority he assigns to numbers. The first substantive chapter in Volume 1, Chapter 2, is on numbers and culture, and it identifies mathematics as a key focus of Spengler's analysis of culture. Since I have discussed Spengler's views at length elsewhere, I will be very brief in identifying the central tenets of his theory.[11]

Number, according to Spengler, is "the symbol of causal necessity". It is the sign of a completed (mechanical) demarcation; and with God and naming, it is recourse for exercising the will to "power over the world". Because Spengler conceives of Cultures as incommensurable (although he allows for the progressive transformation of one Culture into another), he argues that mathematical events and accomplishments should not be viewed as stages in the development of a universal, world "Mathematics". A certain type of mathematical thought is associated with each Culture – Indian, Chinese, Babylonian-Egyptian, Arabian-Islamic, Greek (Classical), and Western. The two major Cultures in Spengler's scheme, Classical and Western, are associated, respectively, with number as magnitude (as the essence of visible, tangible units) and number as relations (with function as the nexus of relations; the abstract validity of this sort of number is self-contained).[12]

Spengler's theory of mathematics yields a weak and a strong sociology of mathematics. The weak form is the one that students of math worlds who have accepted the validity and utility of social talk about mathematics find more or less reasonable. It simply draws attention to the variety of mathematical traditions across and within cultures. The strong form implies the sociological imperative – the idea that mathematical objects are constitutively social.

IV. *The Weak Sociology of Mathematics*

The weak form of Spengler's theory is illustrated by the alternative mathematics discussed by Wittgenstein and Bloor (these are not, in fact, *alternatives* to modern mathematics but rather

culturally distinct forms of mathematics), and by specific mathematical traditions (European, sub-Saharan, Chinese, etc.). The study of these traditions produces stories about the "mathematics of survival", sociocultural bases for the rise and fall of mathematical communities, ethnomathematics, the social realities behind the myths of the Greek and Arabic–Islamic "miracles", and the organizational revolution in European mathematics and science from the seventeenth century on. The episodic history of Indian mathematics can thus be shown to be related to, among other factors, the fragmented decentralization of Indian culture, and the caste system. "Golden Ages" in ancient Greece, the Arabic–Islamic world between 700 and 1200, seventeenth century Japan, T'ang China, and elsewhere can be causally linked to social and commercial revolutions. And the centripetal social forces that kept mercantile and intellectual activities under the control of the central bureaucracy in China can be shown to be among the causes of China's failure to undergo an authocthonous "scientific revolution".

The differences between and within mathematical traditions do not necessarily signify incommensurability. They are compatible with the concept of the long-run development or evolution of a "universal" or "world" mathematics. But the strong form of Spengler's theory – that mathematics are reflections of and themselves worldviews – is another story.

V. *Basic Principles of the Strong Sociology of Mathematics*

There is no ready, intelligible exhibit of the strong form of Spengler's theory of numbers and culture, that is, an example that would persuade a majority of mathematicians and students of math studies that indeed it is possible to describe and explain the content of the "exact sciences" in sociological terms. But there *are* signposts on the road to such an exhibit. Some of these signposts are ingredients of the sociological imperative; some of them are the results of the still very slim body of research in the sociology and social history of mathematics. Based on these signposts, we can begin to anticipate telling a story about mathematics in terms of the strong form of Spengler's theory. The story might begin as follows.

Mathematical workers use tools, machines, techniques, and skills to transform raw materials into finished products. They work in mathematical "knowledge factories" as small as individuals and as large as research centers and world-wide networks. But whether the factory is an individual or a center, it is always

part of a larger network of human, material, and symbolic resources and interactions; it is always a social structure.

Mathematical workers produce mathematical objects, such as theorems, points, numerals, functions and the integers. They work with two general classes of raw materials. One is the class of all things, events, and processes in human experience that can be mathematized (excluding mathematical objects). The second is the class of all mathematical objects. *Mathematical workers* work primarily or exclusively with raw materials of the second class. The more specialized and organized mathematical work becomes, the greater the extent of overlap, interpenetration, and substitutability among mathematical objects, raw materials, machines, and tools. The longer the generational continuity in a specialized math world, the more abstract the products of mathematical work will become.

As specialization increases and levels of abstraction increase, the material and social origins of mathematical work and products become increasingly obscure. In fact what happens is an intensified form of cultural growth. Cultural activity builds new symbolic layers on the material grounds of everyday life. The greater the level of cultural growth, the greater the distance between the material grounds of everyday life and the symbolic grounds of everyday life. Increasingly, people work on and respond to the higher symbolic levels. Imagine the case now in which a mathematical worker is freed from the necessity of hunting and gathering or shopping and paying taxes, and set to work on the purest, most refined mathematical objects produced by his/her predecessors. Under such conditions, the idea that pure mental activity (perhaps still aided by pencil and paper – already a great concession for the Platonist!) is the source of mathematical objects becomes increasingly prominent and plausible. Workers forget their history as creators in the social and material world, and the history of their ancestors working with pebbles, ropes, tracts of land, altars, and wine barrels. Or, because they do not have the language for recording social facts, ignorance rather than failed memory fosters purist conclusions. The analogy with religious ideas, concepts, and thoughts is direct and was probably first recognized, at least implicitly, by Durkheim. In Spengler, this idea achieves an explicit and profound expression.

Specialization, professionalization, and bureaucratization are aspects of the organizational and institutional history of modern mathematics. These processes occurred in earlier mathematical traditions but their scope, scale, and continuity in modern times are unparalleled. Their effect is to generate closure in math

worlds. As closure increases, the boundaries separating math worlds from each other and from other social worlds thicken and become increasingly impenetrable. Specialized languages, symbols, and notations are some of the things that thicken the boundaries around math worlds.

Ultimately, in theory, if the process I have sketched goes on unchecked a completely closed system emerges. This is technically impossible. But as closure becomes more extreme, the math world (like any social world) becomes stagnant, then begins to deteriorate, and eventually disintegrates.

Some degree of closure facilitates innovation and progressive change; extreme closure inhibits them. The advantages of closure must therefore be balanced against the advantages of openness, that is, the exchange of information with other social worlds. Specifically, the danger for the social system of pure mathematics is that it will be cut off from the stimulus of external problems. If pure mathematicians have to rely entirely on their own cultural resources, their capacity for generating innovative, creative problems and solutions will progressively deteriorate. As a consequence, the results of pure mathematical work will become less and less applicable to problems in other social worlds.

The closure cycle reinforces the community's integrity; the opening cycle energizes it with inputs and challenges from other social worlds. The interaction of insider and outsider sociologies of mathematics, mathematical and non-mathematical ideas, and pure and applied mathematics are all aspects of the opening cycle that are necessary for creative, innovative changes in mathematical ideas and in the organization of math worlds.

VI. *The social life of pure mathematics*

Pure mathematics is grounded in and constituted of social and material resources. The idea that pure mathematics is a product of some type of unmediated cognitive process is based on the difficulty of discovering the link between the thinking individual, social life, and the material world. It is just this discovery that is being slowly constructed on the foundations of the works of Durkheim, Spengler, Wittgenstein, and others. Establishing this link involves in part recognizing that symbols and notations are actually "material", and that they are worked with in the same ways and with the same kinds of rules that govern the way we work with pebbles, bricks, and other "hard" objects.

Georga Boole's attempt to discover the "laws of thought" failed because he did not understand the social and material bases of categorical propositions.[13] Categorical propositions are actually high level abstractions constructed out of "real world" experiences, grounded in the generational continuity of teacher–student and researcher–researcher chains which get reflected and expressed in chains of inductive inferences. The "self–evidence" of propositions arises not from their hypothesized status as "laws of thought" but from their actual status as generalizations based on generations of human experience condensed into symbolic forms.

In the case of metamathematics, problems, symbols, and meanings that seem to be products of pure intellect and of arbitrary and playful creativity are in fact objects constructed in a highly rarified but nonetheless social world. Increasingly abstract ideas are generated as new generations take the products of older generations as the resources and tools for their own productive activities; still higher orders of abstraction are generated when mathematicians reflect on the foundations of abstract systems, and self-consciously begin to create whole mathematical worlds.

We can watch this process of moving up levels of abstraction from the "primitive" ground or frame of everyday life in Boole's discussion of the "special law", $x^2=x$. He begins by proposing in an abstract and formal way a realm of Number in which there are two symbols, 0 and 1. They are both subject to the special law. This leads him to describe an Algebra in which the symbols x, y, z, and c "admit indifferently of the values 0 and 1, and of these values alone." Now where does that "special law" come from? Boole actually constructs it on the basis of a "class" perspective grounded in "real world" examples such as "white things" (x), "sheep" (y), and "white sheep" (xy). This establishes that $xy=yx$. He also gets $1^2=1$ by interpreting "good, good men" to be the equivalent of "good men". In the case of $xy=yx$, he argues that since the combination of two literal symbols, xy, expresses that class of objects to which the names or qualities x and y represent are both applicable, it follows that "if the two symbols have exactly the same signification, their combination expresses no more than either of the symbols taken alone would do". Thus, $xy=x$; and since y has the same meaning as x, $xx=x$.[14]

Finally, by adopting the notation of common algebra, Boole arrives at $x^2=x$. We are now back in a world in which it is possible to make ordinary language statements such as "good, good men", and mathematical statements such as $1^2=1$. A careful review of Boole's procedure shows that far from creating a "weird" notation out of thin air, he simply describes a pristine

"everyday" world in which only 0s and 1s exist. This is a rarified world indeed, but it is one in which the rules that govern the behavior of 0s and 1s are similar to the ones that govern the behavior of material objects in the everyday material world. Symbols and notations are simply higher order materials which we work with the same way and under the same sorts of constraints that apply to "hard" materials. (Eventually, Boole interprets 0 and 1 in Logic as, respectively, Nothing and Universe).

It is interesting that there is a tendency for philosophers of mathematics and metamathematicians to reproduce a largely discredited naive realism to "explain" the process of operating on old abstractions and creating new ones. Kleene, for example, writes:

> Metamathematics must study the formal system as a system of symbols, etc. which are considered wholly objectively. This means simply that those symbols, etc. are themselves the ultimate objects, and are not being used to refer to something other than themselves. The metamathematician looks at them, not through and beyond them; thus they are objects without interpretation or meaning.[15]

This process stylizes the idea of objective science. But without a sociological theory of intellect and knowledge, it is impossible to see that the same reasons for abandoning naive realism in physical science are relevant in the case of the so-called exact sciences.

Some mathematicians and philosophers of mathematics have recognized that abstraction has something to do with iteration. They have expressed this recognition in such ideas as "second generation abstract models", and "algebras constructed upon algebras".[16] An important instance of invoking the iteration principle is Richard Dedekind's demand that "arithmetic shall be developed out of itself".[17] One could even claim that mathematics in general is an iterative activity. Sociologically, iteration is the social activity of unbroken chains of mathematical workers, that is, generational continuity.

As generational continuity is extended and closure proceeds in a mathematical community, mathematicians work more and more in and less and less out of their math worlds. As a result, their experiences become progressively more difficult to ground and discuss in terms of generally familiar everyday world experiences. The worlds they leave behind are pictured worlds, landscapes of identifiable things. Math worlds are worlds of symbols

and notations. This is the social and material foundation of so-called "pictureless" mathematics. But mathematical experiences in highly specialized math worlds are not *literally* pictureless. The resources being manipulated and imagined in math worlds are so highly refined that they are not picturable *in terms of everyday reality*; the referents for mathematical objects are increasingly mathematical objects and not objects from the everyday world. Since closure is never perfect, some degree of everyday picturing does occur even in the most abstract mathematical work; and in any case everyday pictures are almost inevitably produced as mathematicians move back and forth between math and other social worlds. At the same time, *new* pictures, math world pictures, are created. During the period that these pictures are being socially constructed (that is, while mathematicians are *learning* to interpret or "see" or "picture" objects in their math worlds), mathematical experience in its more abstract moments will necessarily appear pictureless.

VII. *The End of Epistemology and Philosophy of Mathematics*

The sociological imperative is having an impact on the work of mathematicians and philosophers who utilize the vocabularies of psychology, cultural analysis, empiricism, and pragmatism. But there is some resistance to giving full expression to that imperative. Take, for example, Philip Kitcher's views on the nature of mathematical knowledge. Kitcher, as an empiricist epistemologist of mathematics, constructs a "rational" explanation of beliefs and knowledge that brings psychology into the philosophy of mathematics, but in its psychologistic form. But psychologism cannot carry the burden of his attack on the apriorists unless it is recognized for what it is - a truncated sociology and anthropology. Kitcher seems to realize this at some level. He understands that knowledge has to be explained in terms of communities of knowers, and that stories about knowledge can be told in ways that reveal how knowledge is acquired, transmitted, and extended. This is the *only* story Kitcher can tell; but he is intent on making his story confirm rationality and well-founded reasoning in mathematics.[18]
Rationality and well-founded reasoning cannot be separated from social action and culture. Where it appears that we have effected such a separation it will turn out that we have simply isolated mathematical work as a sociocultural system, and told a sociologically impoverished story of how that system works. The extent to which mathematics is an autonomous social system will

vary from time to time and place to place, and so then will the
extent to which an empiricist epistemologist can construct a
rational explanation for mathematics. "Rational" refers to the
rules governing a relatively well-organizing social activity.
Taking the sociological turn means recognizing that "rational" is
synomous with "social" and "cultural" as an explanatory account.
Kitcher can save the rationality of mathematics only by showing
(as he does very nicely) that mathematics is a more or less
institutionally autonomous activity. But this makes his "rational"
account "nothing more" than a social and cultural account. He
misses this point in great part because he thinks primarily in
psychologistic terms. As soon as we replace psychologism with
the sociological imperative, rationality (as a privileged explana-
tory strategy) and epistemology (as a philosophical psycholo-
gistic theory of knowledge) are nullified.

The same situation is characteristic of philosophical treatment
of knowledge and perception in general which are half-conscious
of the sociological imperative but repress its full power. Richard
Rorty's pragmatism starts out as a strategy for extinguishing
epistemology; but in the end, Rorty "Westernizes" (or better,
"Americanizes") epistemology (this a reflection of the power of
pragmatism as an "American philosophy") and gives it a re-
prieve.[19] He explicitly restricts moral concern to "the conversa-
tion of the West". On the brink of a radical social construction
conjecture on the nature of knowledge, he is restrained by (1)
his stress on the ideal of polite conversation and his failure to
deal with the more militant and violent forms of social practice in
science and in culture in general, (2) his Western bias, mani-
fested in his intellectual and cultural debts, (3) his Kuhnian
conception of the relationship between hermeneutics ("revolu-
tionary inquiry") and epistemology ("normal inquiry") which is
prescriptive, an obstruction to critical studies of inquiry, and
the coup that saves epistemology, and (4) his focus on an
asociological conception of justification. Patrick Heelan also
shackles a potentially liberating contextualist theory (this time
of perceptual knowledge) by talking about Worlds belonging to
the Western community even while he implicates himself in the
project of the "redemption" of science from its Babylonian cap-
tivity in the West.[20]

And finally, I recall Bloor in this context, and his strong
program which helped set the stage for the study of mathematics
by the new sociologists of science. He dilutes the sociological
imperative with a normative commitment to Western culture and
Western science.

Given so much Westernism in philosophy, I have to wonder

about the extent to which philosophers such as the ones I have
been discussing are ideologues of cultural orthodoxy and pre-
vailing patterns of authority. The reality of these sorts of
ideological chains is exhibited in the ways in which philosophers,
pressured by empirical and ethnographic research to see that
Platonism and apriorism (along with God) are dead, reach out
adaptively for social construction. Their failures are tributes to
the professionalization process in philosophy and its grounding
in psychologism. This is all relevant for an appreciation of the
significance of the sociological imperative in math studies be-
cause Mathematics is "the queen of the sciences", the jewel in
the crown of Western science. The protective, awe-inspired,
worshipful study of mathematics is thus understandable –
readily as a vestigial homage to the Western culture, less readily
as a vestigial homage to the Western God. In either case we are
closer to the theology than to sociology of mathematics.

VIII. *Conclusion: Values and the Sociological Imperative*

Math worlds are social worlds. But what kinds of social worlds
are they? How do they fit into the larger cultural scheme of
things? Whose do they fit into the larger cultural scheme off
things? Whose interests do math worlds serve? What kinds of
human beings inhabit math worlds? What sorts of values do math
worlds create and sustain? In his description and defense of
"the sociological imagination" (a form of the sociological impera-
tive) C. Wright Mills drew attention to the relationship between
personal troubles and public issues, the intersection between
biography and history in society, and questions about social
structure, the place of societies in history, and the varieties of
men and women who have prevailed and are coming to prevail in
society. [21] If we approach math worlds from this standpoint, the
questions we ask will be very different from those that philoso-
phers, historians, and sociologists usually ask. The questions I
have posed elsewhere concerning science worlds in general
apply to math worlds too:

> ...what do scientists produce, and how do they produce it;
> what resources do they use and use up; what material
> by-products and wastes do they produce; what good is
> what they produce, in what social contexts is it valued, and
> who values it; what costs, risks, and benefits does scien-
> tific work lead to for individuals, communities, classes,
> societies, and the ecological foundations of social life...

> What is the relationship between scientists and various
> publics, clients, audiences, patrons; how do scientists re-
> late to each other, their families and friends, their col-
> leagues in other walks of life; what is their relationship as
> workers to the owners of the means of scientific produc-
> tion; what are their self-images, and how do they fit into
> the communities they live in; what are their goals, visions,
> and motives.[22]

These questions are relevant to the study of math worlds be-
cause they help us to recover the social worlds that get pro-
gressively excised in the process of producing and finally
presenting (and re-presenting) mathematical objects.
  Explaining the "content" of mathematics is not a matter of
constructing a simple causal link between a mathematical object
such as a theorem and a social structure. It is rather a matter of
unpacking the social histories and social worlds embodied in
objects such as theorems. Mathematical objects are and must be
treated *literally* as objects, *things* that are produced by, manu-
factured by social beings. There is no reason that an object
such as a theorem should be treated any differently than a
sculpture, a teapot, or a skyscraper. Only alienated and alienat-
ing social worlds could give rise to the idea that mathematical
objects are independent, free-standing creations, and that the
essence of mathematics is realized in technical talk. Notations
and symbols are tools, materials, and in general resources that
are socially constructed around social interests and oriented to
social goals. They take their meaning from the history of their
construction and usage, the ways they are used in the present,
the consequences of their usage inside and outside of mathemat-
ics, and the network of ideas that they are part of. The socio-
logical imperative, especially when informed by the sociological
imagination, is a tool for dealienation and for uncovering the
images and values of workers and social worlds in mathematics.

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NOTES

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3. *Grundrisse der Soziologie* , L. Gumplowicz, Manz, Vienna 1905, p. 268; *Genesis and Development of a Scientific Fact*, L. Fleck, University of Chicago Press, Chicago 1979, p. 39 (orig. publ. 1935).
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5. *Local Knowledge*, C. Geertz, Basic Books, New York 1983, pp. 94-120. I draw further on Geertz in the following paragraph.
6. *Art Worlds* , H. Becker, University of California Press, Berkeley 1982.
7. *Knowledge and Social Imagery* , D. Bloor, Routledge and Kegan Paul, London 1976, p. 95.
8. I am grateful to Peter John of the history department at the University of California at San Diego for bringing similarities in the ethical agendas of Spengler and Wittgenstein to my attention.
9. See the discussion of Spengler, Wittgenstein and other contributors to the sociology and anthropology of mathematics in *The Social Relations of Physics, Mysticism, and Mathematics*, S. Restivo, D. Reidel, Dordrecht 1983, pp. 161-175.
10. "Modern Science as a Social Problem", S. Restivo, *Social Problems*, 35 (1988), p. 208.
11. Restivo, 1983, pp. 211-231.
12. *The Decline of the West*, Vol. I, 0. Spengler, International Publishers, New York 1926, Chapter 2.
13. For a more detailed discussion of the ideas introduced in this section, see "The Social Roots of Pure Mathematics", S. Restivo, forthcoming in *Theories of Science in Society,* T. Gieryn and S. Cozzens, eds., Indiana University Press, Bloomington 1989.
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15. *Introduction to Metamathematics*, S. Kleene, Wolters-Nordhoff, Groningen, and North-Holland, Amsterdam 1971, p. 63.
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