Philosophica 35, 1985 (1), pp. 113–134. 

# DIALOGUE LOGIC AND PROBLEM–SOLVING

*Jean Paul Van Bendegem*

## *1. Introduction*

Dialogue logic is a relatively young branch of philosophical logic. Its fundamental problem is to find a set of rules that, when rigourously applied, will result in correct dialogues and to provide a justification for this set such that correct dialogues must result. The initial focus was on the former problem — see, e.g. Paul Lorenzen's initial formulation — whereas today the latter problem attracts all attention. This volume itself is a witness to this state of affairs. My own contribution will be no different.

I will deliberately restrict myself to a particular context, namely a problem-solving context, and try to show how this context generates a justification for the rules of the dialogues. The first claim of this paper is that, in the problem-solving context, the dialogue rules are of a particular form. I believe this form to be typical for dialogues. Hence, it allows us to make a distinction between monologues on the one hand and dialogues on the other hand. This solves an important problem: in most proposals dialogues are indistinguishable from monologues. Even if a monologue can be interpreted as a dialogue with an imaginary partner, nevertheless, it is still a monologue.

The second claim is that the arguments, sustaining the first claim, involve economical considerations. This may sound (rather) surprising, but in a problem-solving context, given that the problem-solver has a limited budget at his disposal, such considerations appear quite inevitable and natural. Hence my previously mentioned restriction to problem-solving. Although I believe that the model presented here, can be extended, I will not do so. In paragraph 2, the model is presented and in paragraph 3, various aspects of the model are discussed.

*2. Presenting the model.*

Consider two problem-solvers. Why should they communicate with one another? The answer can only be: because of efficiency. As an example, take two mathematicians. Suppose the first is working in number theory, the second one in complex function theory. Many interesting links have been established between number theoretic propositions and complex functions. Suppose the first mathematician is investigating a problem and he discovers that some theorems about complex functions are relevant to his problem. It is obvious that he will save a lot of time by asking the other mathematician what is the precise content of these theorems and what can be derived from it. He will save a real lot of time by just accepting what the other mathematician says. Although this would reduce any dialogue logic to the trivial rule: "If X says p, accept p", clearly this rule runs counter to the efficiency requirement. For, if the first mathematician encounters a problem with the accepted results, he will be forced to ask the other what the problem could be. The trivial rule alone, obviously, is insufficient (and inefficient). But it is equally obvious that the other extreme suffers the same shortcoming: if the first mathematician asks the second one for a complete derivation of the theorems in question, he might as well have studied complex function theory himself without the need to bother his colleague. In this case the dialogue can no longer be called a genuine dialogue: at best, it is a mimicking of a monologue.

A more likely story is that the number theorist is satisfied with a partial explanation. He does not need a full derivation. What he needs is a derivation that enables him to search for a possible error, if it occurs. This feature of dialogue logic is much too often overlooked and neglected. I take this to be a serious flaw, as I believe that precisely this feature can distinguish genuine dialogues from "monologues-in-disguise".

First, I will present a set of rules that govern the behaviour of a problem-solver. These rules are referred to as basic and derived capabilities. Basic capabilities are procedures a problem-solver can always execute. Derived capabilities are procedures that allow a problem-solver, when presented with some procedures, to generate new procedures. Each of these capabilities will be presented and motivated. Dialogues are introduced and a particular interpretation is given to the concept of partial explanation.

*Basic capabilities*

The list presented here is not exhaustive. However, it is certainly minimal, in the sense that a problem-solver, lacking any of these, is seriously handicapped. He is not really reduced to nil as a problem-solver, but the limitations imposed are so severe that few interesting problem solutions will follow. I must add here right away that some concessions to existing logic are made, such as the use of the classical connectives "&" and "v". I fully realize that they can be questioned, but it cannot be denied that these connectives play a central role in logic. In other words, I do not claim that they are the basic signs to deal with, but rather that they must be included in any discussion of problem-solving logic. The model is such that other signs can be incorporated.

The notation used throughout is fairly simple. All formulas are of the form :

$$(p?;H)q.$$

Its reading is: if p is the case, then, if H is executed, q will result.[1] p and q are formulas, involving no procedures such as H, and I assume that procedure-free formulas can be combined into p&q and pvq. If no test, such as P?, is involved, the formula is shortened to (H)q. It is understood that H itself does not involve tests of any kind.

(1) $(p?;\text{store})p$

This expresses the capability of a problem-solver, henceforth called X, to remember something. Hence, the choice of the name "store" for the procedure in question. I assume here that X has perfect memory. This is not to say that the restriction that X's memory is finite, is a futile one, quite on the contrary, this will force X to a continuing "reshuffling" of his memory's content and to use devices such as books and the like to extend his memory. But for the minimal model, we neglect this feature, that can be incorporated in an extension of the model. Having a memory is a basic capability that need not be argued for. It is sufficient to look at cases of damaged memory to see how seriously one is disadvantaged even by a reduction of memory[2].

(2) $(p\&q?;\ \mathrm{select}_L)p$ and $(p\&q?;\ \mathrm{select}_R)q$

If X is presented with a composite piece of information such as p&q, he must be capable of decomposing this piece of information. Again, it is hard to imagine a problem-solver deprived from this capability. Of course one could argue that there is no necessity to suppose a left-right symmetry. For, if X is capable to select the left-most element, a repeated application of this selection will allow him to arrive at the right-most element. E.g. :

$$\begin{array}{ccccc} p\&q\&r & \longrightarrow & q\&r & \longrightarrow & r \\ & \searrow & p & \searrow & q \end{array}$$

But this supposes that X can decompose a piece of information such as p&q&r into two parts, viz. p and q&r, that X can store both elements and continue to work on either of them. Here again we suppose a form of symmetry, because p and q&r can be treated in the same way. The argument could be put forward that this capability presupposes that p&q does not form a whole (in some wholistic sense of the word). Of course, if we intend the sign "&" to carry an intensional meaning as well, the analysis becomes more dificult. But whether one views the world wholistically or not, it seems inevitable that X must be able to decompose a piece of information. Of course, intensional signs will play a vital role in extensions of the minimal model. Rather the point is that "&" in its extensional meaning, must be among the signs required for efficient problem-solving.

(2) $(p?;\ \mathrm{altern}_R)p \vee q$ and $(q?;\ \mathrm{altern}_L)p \vee q$

If X has some piece of information, p, then he must be capable of adding an alternative to it, i.e. (in a quite general meaning) to introduce a choice. One might wonder what the efficiency of such a procedure could be. But imagine that X is involved in some problem-solving situation and that he has found that $x>0$. And suppose that this piece of information is relevant to another problem where the information $x\geqslant 0$ is interpreted as $x>0$ or $x=0$). It is then very efficient if X can say: since $x>0$ is available, then surely $x\geqslant 0$ is available as well. But one might remark that this capability is at the same time very inefficient, because it leads to an information explosion. For, if X has the information "$x>0$", then he can add to it all sorts of expressions, such as "$x>0$ or the moon is not a star", or "$x>0$ or I have a bicycle", and so on. But, as we will see, there is a restriction on this explosion. Adding information is only relevant inasmuch as some problem-solver can do something with it. Similar remarks on symmetry and extensionality can be made here.

(1)–(3) sum up the basic capabilities of X as a problem-solver. As said before, I do not claim that this (short) list is exhaustive, but I do claim that it is minimal.

*Derived capabilities.*

Basic capabilities are clearly not sufficient. The only thing X could do up to this point is, when presented with a piece of information, to perform a procedure, using it, and stop. What X needs are rules that allow him to generate new procedures. If a formula (p?;H)q is presented, then its testfree companion is (H)q. Note that for the rules, we do not restrict ourselves to basic capabilities. X must of course be capable of dealing with all sorts of procedures, including the basic ones.

(1) $$\frac{(p?;H)q\ ,\ (q?;H')r}{(p?;H;H')r}$$

If X can transform p into q and q into r, then by joining these two procedures after one another, he can transform p into r. A correct name for this rule is *diachronic composition*, since it involves a sequential composition of procedures, I believe that this rule hardly needs any motivation. Its test-free companion is:

$$\frac{(H)\ q,\ (q?;H')r}{(H;H')r}$$

Obviously the test q? cannot be left out, for then the rule becomes redundant. It then only states that if r can be obtained by H', it can also be obtained by doing something else first and then by executing H'. Clearly a waste of time and energy !

(2) $$\frac{(p?\ ;\ H)q\ ,\ (p?\ ;\ H')r}{(p?\ ;\ (H||H'))q\&r}$$

The sign "||" indicates that H and H' are executed concurrently.

Its test-free companion is :

$$\frac{(H)q\ ,\ (H')r}{(H||H')q\&r}$$

These are the counterparts of the basic capability (2). These rules allow X to put pieces of information back together again. This rule and its companion may be called *conjunctive synchronic composition.*

(3) $$\frac{(p?\ ;\ H)r\ ,\ (q?\ ;\ H')r}{(pvq?\ ;\ (H/H'))r}$$

Corresponding to basic capability (3), we find the rule of *disjunctive synchronic composition.* The sign "/" now indicates that H or H' is executed, depending on what is available. This rule, the counterpart of information addition, generates the possibility to use a weakened piece of information to obtain something, viz. r. Of course additional information must be presupposed, in this case the knowledge that p, as well as q, generates r. This explains the comment that disjunction does not lead to an information explosion. For only if both disjuncts lead to something, can they actually be used to obtain something. An interesting feature of this fact is that, given a disjunction such as "x>0 or the moon is not a star", we cannot tell whether this disjunction is interesting or not. For suppose that from x>0 we can derive that the universe must contain an object that is not a star — suppose that the x in x>0 is a variable occurring in an equation about the distribution of matter in the universe — and that from "the moon is not a star" we can derive "there is an object that is not a star". Then we can derive that "if x>0 or the moon is not a star, then there is an object that is not a star". Clearly in this case, the disjunction is interesting. But under the same circumstances, it is equally clear that "x>0 or God exists" is very likely to be of little value.

Suppose that X and Y are two problem-solvers as presented above, then the question is what a dialogue between them could look like. A classical approach would be the following. Say X has found a solution for transforming p into p&p :

$$(p?\ ;\ (\text{store}\ ||\ \text{store}))(p\&p)$$

Y can question X about the correctness of this solution, e.g. by assuming p and then asking X how he could arrive at p&p. In this type of dialogue, Y will receive full information, because after the dialogue is finished, Y has all the ingredients to reconstruct the procedure that was the subject of the dialogue. But another type of dialogue is possible. Let us turn back to the example of the two mathematicians. Suppose the complex function theorist says that "the product of a complex number $x+iy$ and its complex conjugate $x-iy$ is $x^2+y^2$". In terms of our approach, this means that the number theorist says that "if $z=x+iy$ and $z=x-iy$ then I have a construction such that $zz=x^2+y^2$". That is, he does not give the construction itself, he only says that there is one. The advantage of the logic presented here, is that there is a method to reflect this important distinction. If X has a procedure H such that (p?;H)q holds, then either he can present in a dialogue this statement itself, or he can only state that he has found a procedure. This last statement I will represent by

$$p \rightarrow q$$

The connection between (p?;H)q and $p \rightarrow q$ is straightforward :

$$p \rightarrow q \text{ iff } (p?;H)q, \text{ for some } H. \quad (*)$$

Stated thus, the left hand-side does not contain anything new. But from a dialogue point of view, this is not so. For suppose X claims that $p \approx q$. In that case Y does not know what the procedure is that makes it possible to transform p into q. Of course one strategy for Y could be to ask for the procedure, i.e. the H such that (p?,H)q. This reduces the dialogue to the former type mentioned. But there is another possibility. Let me illustrate this with the following example: suppose X claims that $p \rightarrow q$. And suppose that Y knows that $r \rightarrow q$. It makes (a lot of) sense for Y to ask X if $p \rightarrow r$ is a part of his procedure. It is important to realize that Y does not need to know what the precise structure of the procedure involved in $p \rightarrow q$, is. Suppose X says this is the case. The dialogue is now reduced to the problem $p \rightarrow r$. This type of dialogue is much closer to daily practice than e.g. the Lorenzen-type of dialogue. Take e.g. this dialogue :

X : "A second-degree equation with negative discriminant has two complex solutions such that their product is a real number".

Y : "Why is that?"

X : "Because the solutions are conjugate".

Y : "I see".
Y "sees" it because he knows that the product of two complex conjugate numbers is always a real number and (apparently) because he knows that a second-degree equation with negative discriminant has complex conjugate solutions. There is no need in this dialogue to actually write out a proof for this property.

The distinction between these two types of dialogues can be formulated thus :

(a) the former dialogue (related to the classical approaches) is a *bottom-up dialogue.* If X claims that $p \rightarrow q$, then Y asks for the procedure H such that $(p?;H)q$. Y accepts p and then X must show how p can be transformed into q by stating all the primitive procedures involved one by one.

(b) the latter dialogue is a *top-down dialogue.* If X claims that $p \rightarrow q$, then Y asks for *a* decomposition of this procedure, not necessarily in primitive parts, e.g. X may answer that $p \rightarrow r$ and $r \rightarrow q$. If it turns out that Y knows that $p \rightarrow r$, then this part of the dialogue is terminated and the dialogue now focuses on $r \rightarrow q$. There is no need — unless Y's knowledge is minimal — to go down to the bottom-level. Of course several decompositions may be tried out, a common experience in dialogues. Note too that if Y knows that $p \rightarrow r$, then it is not necessary at all that Y has the same procedure in mind as X. Example : suppose the statement is

$$p \rightarrow (p \& p) v p$$

For X, this may stand for :

$$(p?; \mathrm{altern}_L)((p \& p) v p),$$

and for Y, this may be :

$$(p?; (\mathrm{store} \,||\, \mathrm{store}); \mathrm{altern}_R)((p \& p) v p)$$

If the rule (8) is extended such that

$$p \text{ iff } (H)p, \text{ for some } H.$$

then all capabilities can be easily transcribed :

| | |
|---|---|
| 1. $p \to p$ | 1. $p \to q,\ q \to r$ / $p \to r$ |
| 2. $p\&q \to p$ | 2. $p, p \to q$ / $q$ |
| 3. $p\&q \to q$ | 3. $p \to,\ p \to r$ / $p \to q\&r$ |
| 4. $p \to pvq$ | 4. $p, q$ / $p\&q$ |
| 5. $q \to pvq$ | 5. $p \to r,\ q \to r$ / $pvq \to r$ |

Except for one missing axiom, $p\&(qvr) \to (p\&q)v(p\&r)$, this is exactly the logic $P^+$ of Arruda and da Costa.

The dialogue rules can be written down quite easily :

(a) X makes a claim — which can either be of the form B or of the form $A_1, \ldots, A_n/B$ where $A_i$ and B are formulas of the type p or $p \to q$ — and Y accepts to control the claim. Between brackets the knowledge of X and Y is added,

(b) if a premise of a basic capability is encountered, Y accepts it. If Y has already some knowledge (between brackets) and some item of this knowledge is encountered in the dialogue, then Y accepts it,

(c) corresponding to each of the derived rules, there is a dialogue rule. I present here one case, as the other cases can be easily derived. Corresponding to

$$p \to q,\ q \to r\ /\ p \to r,$$

We have the following rule : Y asks D? (D stands for diachronic composition). X answers "yes" if B has the form (p?; H; H')q and the dialogue is split in two: one for the case (p?; H)r and one for the case (r?; H')q, i.e. X says $p \to r$ and $r \to q$. X answers "no" in all other cases and Y can ask a new question. For the other rules, the corresponding questions are: Df? (f stands for test-free), CS? (conjunctive synchronic), CSf? and DS?

(d) Y has lost the dialogue if he has accepted all cases. In all other cases he has won.

A typical example of a dialogue (in which Y has no knowledge) :

| X | | Y |
|---|---|---|
| [(p?;H)q,(p&q?;H')r, (p?;(store\|\|H);H')r] | | [–] |
| (1) $p \to q,\ p\&q \to r/p \to r$ | | D? |
| (2) yes | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| (3) | $p \to p \& q$ | | $p \& q \to r$ | D? | | Acc |
| (4) | no | | | CS? | | |
| (5) | yes | | | | | |
| (6) | $p \to p$ | $p \to q$ | | Acc | Acc | |

In step (3), in the left-hand columm, Y has asked a bad question. In the worst case, he could have made four bad selections. But eventually he must select the "right" question, (unless, of course, X cannot answer any of the questions, but then Y wins).

The important feature to note is that Y knows that the claim of X is correct, although at the end he has no knowledge whatsoever about the procedures H and H'. And that is precisely what I mean by a partial explanation. It is not necessary at all for Y to go through the whole procedure X has found. And in this sense, this type of interaction between X and Y constitutes a genuine dialogue. For suppose that the dialogue above is such that X and Y coincide, then the dialogue becomes a monologue. But then Y could as well look at the procedure itself, since it belongs to his knowledge. No need to ask himself questions of how the procedure is structured, for he can simply look at it. But if X and Y are separate entities — or formally speaking, if X has no direct access to the knowledge of Y and vice versa — then the questioning dialogue becomes an interesting tool for Y to control X's claim. Furthermore, it is a type of interaction that makes a dialogue worthwhile: Y can gain knowledge without having to go through all the details. And that is why we talk to one another: because some one else may know something I do not and through the dialogue I can save time and energy. I do not have to reconstruct the other's solution, I can check it in an economical way. Of course, X could simply present the procedure to Y, but then the dialogue is redundant. Y can go home and check the procedure by himself. In a sense, X and Y are then equal, because they have the same knowledge. But in the example presented here, at the end of the dialogue, Y can still leave the brackets empty at the top (although he could at the end of the dialogue construct a procedure), although he knows that the claim is correct.

To end this paragraph, let me reformulate the two claims of this paper. The first claim was that the rules of a dialogue are of a particular form: they are the rules governing the *top-down analysis* of a claim made by X. These rules are directly derived from the capabilities of X and Y, seen as problem-solvers. As to the second claim, it is now clear that, only in a genuine dialogue context, does a top-down analysis represent an economical profit for Y. In the monologue case, this is not so. In other words, if no such profit were possible, there would be no need to conduct a dialogue. For why do problem-solvers communicate: because of efficiency.

*3. Discussion and comments*

The case of the missing axiom. As said, the logic presented here differs from P because the axiom

$$p\&(q\vee r)\rightarrow(p\&q)\vee(p\&r)$$

is lacking. The reason to do so is quite simple. There is at least one problem-solving context in which this axiom is rejected, viz. quantum logic. Therefore the axiom was not added to the list of basic capabilities. The other capabilities are valid in that context as well and, to repeat, it is hard to see in what context they could indeed be violated. This is not to say of course that in specific contexts, the axiom may be added to the basic capabilities. This leads in a straightforward manner to the question whether the logic $P^+$ can be extended into, say, classical propositional logic. There are reasons not to do so.

Extending $P^+$ to classical propositional logic would require the introduction of negation and the introduction of nested implications. As to the first, I will assume that negation can be introduced in a standard manner, i.e. by adding the axiom $p\vee\sim p$ and the necessary rules to obtain classical logic. The problem obviously is what meaning we can attach (in this context) to a negation. What does it mean that a problem-solver comes up with $\sim p$. Several options present themselves :

(a) it can simply mean that the problem-solver constructs a solution that ends with the message $\sim p$, e.g., "this equation has no square roots". But then the well-known intuitionistic criticism applies and there we have at least one reason to reject classical logic as a candidate for a dialogue logic. This point need not to be stressed any further.

(b) it may mean the failure to obtain p. But then we have a negation on the meta-level. Although it is tempting to state that "if one fails to obtain p, then $\sim p$", this is surely unacceptable, because of the criticism sub (a).

(c) of course, why not simply use the intuitionist's version itself. $\sim p$ is the case iff $p \rightarrow f$. But then surely, $p \vee \sim p$ is not acceptable. So, perhaps not classical logic, but intuitionistic logic is a probable candidate. But there are still some problems left. And they are not easily dismissed. Practically all versions of intuitionist logic[3] accept the so-called "paradoxes of implication" either as axioms or as theorems, i.e.

$$p \rightarrow (q \rightarrow p)$$
$$(p \& \sim p) \rightarrow q$$

As to the first, it is necessary to attach a meaning to nested implication. Using the transcription rules

$$p \rightarrow (q \rightarrow p)$$

becomes

$$(p?;H)\ (q \rightarrow p)$$

Clearly, $q \rightarrow p$ cannot be read as an actual construction, but as a message informing us about a construction. I will use a Gödel- like notation to express this feature :

$$(p?;H)\ (\ulcorner (q?;H'p \urcorner)$$

What does this formula state? It says that if X has p then he can construct the message that states that if q is the case, p can always be obtained. And this should always be the case. But is it? If I say that X has p to start with, then it is obviously so that X will have p at the end. He can simply store it in his memory. But then, what is q doing there? If the statement is formulated as a rule, the problem is even more obvious:

$$p/q \rightarrow p$$

is than transcribed into

$$(H)p/(q?;H)p$$ [4]

This states that if p can be obtained in a test-free manner, then it can also be obtained if a test is added. But this is close to being wrong. For if q is something like "there is a row of nine sevens in the decimal development of $\pi$", then (q?;H)p will not start, so I don't even know whether p will result. If one then says that this is no problem, because we already have p to start with, then clearly the role of q is superfluous. Compare these comments to the corresponding dialogue situation.

| | X | Y |
|---|---|---|
| | [(H)p,(q?;H)p] | [–] |
| (1) | $p/q\rightarrow p$ | D? |
| (2) | no | Df? |
| (3) | no | Cs? |
| (4) | no | Csf? |
| (5) | no | Ds? |
| (6) | no | |

Y has won the dialogue, because X was unable to answer any of the questions. Therefore Y does not have to accept the claim of X. In sort, the dialogue rules do not "recognize" the derivation from p to $q\rightarrow p$ as a genuine construction, since q has little to do with p. Of course this criticism is well-known. It is precisely the starting-point of the study of relevant logics.

All this is hardly new and the present author is well aware of this fact, but what is interesting, is that the context proposed here, viz. problem-solving supports these ideas. To quote the famous example of Anderson and Belnap, there is nothing wrong with the mathematician, who, after proving a theorem T, states that "if Fermat's Last Theorem is the case, then T" is true as well. He is just wasting his time ! Such a statement in a dialogue would reduce the economical advantage of that dialogue because Y will never get an answer to his questions, as there is no construction that leads from FLT to T. A rule that would say: "if p can be obtained test-free, then, adding a test to this construction, does not affect p", sounds very uneconomical indeed.

The same critique applies to $(p\&\sim p)\rightarrow q$. This implies that if Y finds a contradiction (or a falsehood) in X's construction, the dialogue is finished. But that is too severe. Most interesting dialogues are precisely those in which a contradiction is found and in which both parties try to do something about it. In the dialogue context outlined, such a thing is possible. At the start of the dialogue it was not requested that X's construction is a correct one. Y may discover a mistake in it. In the best of cases, locating an error is a very efficient procedure to fix it. After all, that is precisely what computers do. If there is an error in your program, the computer — if it is smart enough— will inform you that there is a "SYNTAX ERROR IN LINE n". This message is an invitation to repair the program, not to abandon it !

The conclusion must be that the minimal logic P can indeed be extended, but not beyond intuitionistic logic. That is to say, it can be done, but it is just extremely uneconomical to do so. Now this still leaves an impressive realm of possible logics: relevant logics, paraconsistent logics, dialectical logics and the like. I must leave it here as an open question whether or not one (and just one) of these logics can be singled out as the most economical one.

An important criticism that may be formulated against this proposal, is that one has to suppose that X possesses a detailed description of the procedure(s). Only in that case are we guaranteed that the dialogue will end. (Y can only ask a finite number of questions about a finite structure, the procedure, hence the dialogue must end in a finite number of steps). But the advantage of, say, Beth's semantic tableaux method, is precisely that we can still decide the truth of a claim, without having to know the actual proof. It is true of course that, if X has no detailed procedure (the formal equivalent of a proof) then there is no guarantee that the dialogue will come to an end. But on the other hand, if X has indeed no complete knowledge, it is obvious that he cannot answer all questions. But, I hear the critic say, in that case Beth's method works. I believe a distinction should be made here. What I try to do in this paper, is to show what makes dialogues in a certain context, typically dialogues. This implies that the first problem on my list is not to device such a logic that is known to be semantically decidable (for otherwise Beth's method fails too!). If it turns out that the dialogue method here is indeed decidable, so much the better. But it was not the starting point. Furthermore, Beth's method can be incorporated in the type of dialogues constructed here. Suppose that X has checked a statement $A_1, \ldots, A_n/B$ by some tableaux method. Then there is nothing wrong if X considers this tableaux construction to be the procedure that allowed him to construct B, starting with $A_1, \ldots, A_n$. Of course the rules will have to be adapted to this case, but nothing prevents Y to ask questions (in a top-down fashion) about the semantical method that X used to sustain his claim. This argument helps in clarifying the difference between my approach and Beth-like approaches. I believe there can be a genuine dialogue about Beth-tableaux, but not that the tableaux method itself is a dialogue. For if a tableaux is written out in detail, one will see that Y does everything that X could have done. In other words Y must go through all the details of the claim of X. But, on the basis of my efficiency criterion, this is not a dialogue. Y could as well take X's claim with him and check it at home, using the tableaux rules. One might remark that if Y's knowledge is nil up to the basic and derived capabilities, then the method presented here will require as much time as, say, Beth's tableaux method. True, but in actual dialogues, this is very often not the case. Practically all scientific communications fall in this area. In classical dialogues, Y's role is reduced to the minimum. He doesn't need to have any knowledge of his own. And even if he did, there would be no way to use it in an interesting and economical way. Precisely such an active interrogative method as outlined here makes it possible for Y to put his knowledge at work. X and Y are partners in the dialogue. A dialogue cannot be an interaction in which one party has to inform the other party what to do next. And reasoning once again from an economical point of view, what makes it interesting to put two individuals together? Simply the fact that two sources of knowledge are "linked" to one another. And who is denying that a group of problem-solvers can do more than an individual problem-solver?

If matters become more complex, interesting features can be deduced from this frame-work. Suppose that the problems become so complex that X himself is no longer capable of writing out the solution in full detail. Evidently classical methods will fail to give clear-cut answers. Some questions asked by Y, may be answered, some not. Instead of logical rules, heuristics seem to be a more appropriate tool. That is, after some questioning, Y will accept the answer or not. In Van Bendegem (1982) I have developed these consequences in more detail for the mathematical realm. It is startling to see that in mathematical practice diverse methods are used that are not deductive at all. And that for the simple reason that, if written out in full detail, the proofs become so complex that they are no longer accessible to an individual. Furthermore, it may be the case that X and Y use different extensions of P. Suppose X is a classical logicist (and thus has no problem with statements such as $p \vee \sim p$) and that Y is an intuitionist. If X now claims that "either FLT or $\sim$FLT" then Y will ask for the construction that guarantees that and if X would say "because it is an axiom", then Y will question the axiom itself. In the classical dialogues such situations cannot occur, but the frame-work presented here, does allow that.

A problem I do not consider, is the search for a solution. The dialogue starts if X has a solution. I consider the problem of how a discovery is made to be of a different nature. We are still far away from a logic of discovery (if such is possible at all). Thus the best we can do at the moment is to use heuristics. I do not deny — practice is the best counterargument — that dialogues are of crucial importance in that area as well. I do claim that they are of a different nature. If Y were to ask questions to X along the lines outlined above, all answers will be of little value, as X does not have a solution (yet). In this case, it will be of more importance to search for methods that allow to decompose the problem into simpler units, that can be handled by two or more problem-solvers. In short, in this context, the most economical procedure is to turn Y into a companion of X in his search for solution. The kind of information they will exchange, will depend largely on the methods for searching a solution. Hence in this case, the dialogues will depend on the discovery heuristics. And these are radically different from the logic of procedures. An important implication is that I do not believe that a single non-trivial frame-work can be presented for all dialogues. To give another example, it would be ridiculous to analyse the dialogues in e.g. Samuel Beckett's "Waiting for Godot" with the method outlined here. It follows from these remarks that I am rather sceptical about the maxim-approach, i.e. the attempts to formulate general rules, applicable to all dialogues, guaranteeing a speaker, that, when following these rules, success is likely to come about. Only if the context of the dialogue is more detailed, is it possible to derive some features relating to the rules of the dialogue. Thus the maxim "Be relevant!" makes a lot of sense in the present context, but if, e.g., in a political discussion, the objective of one party is to conceal that they made a mistake, it is very likely (and perhaps required) that they should ignore the maxim.

An important feature of these dialogues is that truth is not a central notion. If X presents his solution, X does not need to know that the solution is true. Since the core of the solution is a procedure, it does not even make sense to say that the procedure is true. It is better to say that it is correct. So at the beginning of the dialogue we do not have to assume that the procedure is correct. If Y asks questions to X, according to the rules, it may very well happen that at a certain point, Y accepts an unstated procedure

$p \rightarrow q$, because he has encountered it sometime before. But it may very well be that the procedure is not correct. Only if Y would go through the procedure in full detail could he have a guarantee of the correctness of the procedure. But on economical grounds, this is an unreasonable requirement. Of course, one might claim that truth is indirectly present, because the following definition could be stated :

> $(p?;H)q$ is true iff the procedure H acting on p and resulting in q is correct.

But this is not "true" in its standard meaning. For it may very well happen that the procedure turns out to be incorrect. Then $(p?;H)q$ would be false and knowing something to be true is then as relative as knowing that something is correct. In other words, the truth of $p \rightarrow q$ does not guarantee us in any way that $p \rightarrow q$ will remain true. But a time-dependent truth notion is of little interest. It certainly does not force us to suppose any relation whatsoever between the statement and the world. With truth absent, dialogues in which a change occurs — a statement $p \rightarrow q$, believed to be correct, turns out to contain a mistake — can be treated in this frame-work. Such is often the case in mathematics. A proof is accepted as correct and only at a later stage is an error found in the proof or the construction. In standard logic such a case cannot be dealt with, but in the present frame-work this presents no problem. It may even be the case that an incorrect procedure generates correct results. The best known case in mathematics is the Dirac $\delta$-function, which was being used by physicists — thus accepted as correct — while mathematicians were searching to remove the inconsistency[5]. This would imply that in terms of classical dialogue schemes a dialogue about the $\delta$-function would be utter nonsense, and that it must be rejected. But the course of events in scientific practice does not follow the neat logical scheme: statement-discussion-acceptance or rejection. So either these cases must be considered infortunate events in scientific practice, or, one tries to device a frame-work in which they can be dealt with. I take it that the proposal made here meets that requirement.

Besides the context of scientific practice, there is at least one other area where evidence can be found for this model of dialogues in problem-solving context and that is in artificial intelligence. As a concrete example the program for the Blocks World can be used.

The machine can perform a set of actions, such as PUT—ON (putting a block on top of another), GRASP object, UNGRASP object, and so on. The whole procedure to put e.g. a block B on top of a block C can be represented by a tree :

PUT—ON B C

PUT—AT B (coordinates)

GRASP B    MOVE—OBJECT (coordinates)    UNGRASP B

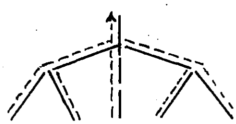
CLEAR—TOP B    .....    ...

GET—RID—OF A

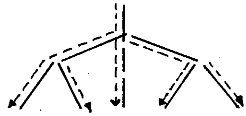
...

If a how-question is asked to the machine at a certain level — say, we ask, "How do you PUT—AT B" then the machine answers by moving down one level: "I first GRASP B, then I MOVE—OBJECT and finally I UNGRASP B". This corresponds completely to Y's question if there is a diachronic decomposition of the given procedure.

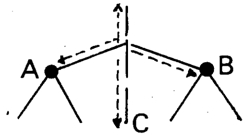
It is interesting to note that another type of question can be asked as well, namely a why-question. This is answered by moving up one step in the tree. So, if it is asked why B is grasped, the answer will be "Because I want to PUT—AT B". This suggests that the method outlined here may work in both ways. Suppose Y would ask: "does the procedure involve H and why?" Then the answer will again give partial information about the complete procedure. So instead of trying to find a procedure that Y knows, he may speed up the process by actually asking if the procedure is a part of the global procedure or not. In this case too, it is not necessary that Y knows the total procedure to judge its correctness. The overall picture is this : instead of going bottom-up



or going top-down



Y searches for a node somewhere in the tree and tries to find his way in the tree




At points A and B, Y does not need to go any deeper because the procedures at those nodes are accepted by him, although it may happen that sometimes he has to go down all the way (such as in C). The diagrams themselves are excellent proof of the economical advantage of this approach.

A last aspect to be mentioned is the possibility to extend the economical features of the dialogues. In the basic model the economy consists in minimizing Y's costs to judge the correctness of the solution. A possible extension might be the following. The model does not attach an economical value to the statements themselves. Suppose that Y has to pay for the information obtained. If e.g. he asks if the procedure has a top-level diachronic decomposition, X may answer that the answer to that question costs x. If Y is willing to pay that sum, the answer can be bought. If Y has a finite budget at his disposal, he will try to minimize the cost of the answers. The problem becomes more complex : at what cost can I find out the correctness of the procedure. That this problem is not absurd can be judged by the following example. When Appel and Haken presented their proof of the four-colour theorem, many mathematicians rejected it because it involved the use of a computer that had to colour some two hundred thousand maps. It was rejected because the computer part of the proof was not accessible (in detail) to humans. In our model, this is equivalent to stating that no questioning of Y can guarantee full correctness. Even if Y accepted to go all the way down to the bottom, this is impossible because of the time Y should need to do so. But even if Y did accept the computer proof, a control of the proof would require the use of a computer. But computer time must be bought in the real sense of the word. So it may still be the case that the proof is not accessible to certain Y's because the answers to be obtained are too expensive.

This is not a shocking result to someone who is familiar with the subject of the economy of information. The claim in this paper is that these economical considerations can be extended to logic itself. As an extreme example, one could consider a dialogue logic with secrets. A secret is a piece of information such that the cost of obtaining that information exceeds the budget of the questioning party. As a devoted Sherlockian myself, I must make a side-remark here about Jaakko Hintikka's work on Holmesian logic. His preferred example is the barking dog, but in many cases Holmes is dealing with secrets. In many of Watson's reported cases Holmes knows the guilty party, but questioning that party would not help him much. Instead he must try using his celebrated methods to obtain information (without the need of the guilty party) that may establish the correctness of his conclusions. Even in the incident of the barking dog, suppose that all witnesses had answered that the dog did bark (although he did not), then Holmes must first deduce they are all lying. This does not invalidate Hintikka's proposal — on the contrary, as must be clear from the model, I share with him the insistence on the importance of well-directed questions — but it does imply that important facets of dialogues are not dealt with in his account.

## *4. Conclusion*

The material presented in this paper, must be seen as part of a larger project, namely an economy of logic. The basic underlying motivation is simple enough. Since information is an economical good such as any other, and dialogues are a special kind of information exchange, dialogues must have economical features. Hence one might expect — and this is the point I have tried to develop here — that the rules of dialogues too are subject to economical considerations.

As I believe this approach to be rather novel, I have concentrated on elaborating and discussing the model in some detail. Apart from occasional remarks, I have not tried to compare my model with other existing proposals for dialogue logic. This would require a separate paper, again a matter of economics.

## NOTES

[1] The notation is borrowed from dynamic logic. See e.g. Harel (1979). However the treatment here is different, so that the existing semantics for dynamic logic cannot be used here.

[2] See e.g. Lindsay & Norman (1977).

[3] Excluding, of course, those intuitionists that reject the use of formal logic.

[4] For the sake of simplicity, the same H is used. Of course, a more general transcription could be used : (H)p/(q?;H')p. The argument remains unchanged, but becomes just longer.

[5] The inconsistency is the following. The definition of the Dirac $\delta$-function is :

$$\delta(x) = \begin{cases} 0 \text{ if } x \neq O \\ \infty \text{ if } x = O \end{cases}$$

$\delta$ must satisfy one condition : $\int_{-\infty}^{+\infty} \delta(x)dx=1$, but this is impossible because the integral is not defined for infinite values, hence it is not defined for x=O.

## BIBLIOGRAPHY


ANDERSON, Alan Ross & BELNAP, Nuel D., Jr.: *Entailment, the logic of relevance and necessity*, volume 1, Princeton University Press, Princeton, 1975.

APOSTEL, Leo: "Towards a general theory of argumentation" in: BARTH, E.M. & MARTENS J.L. (eds.): *Argumentation, approaches to theory formation*, John Benjamins, Amsterdam, 1982, pp. 93–122.

ARRUDA, Ayda I. & da COSTA, Newton C.: "On the relevant systems P and $P^+$ and some related systems", *Realtorio Interno no 174*, Campinas, Brasil, 1980.

BARTH, E.M. & KRABBE, E.C.W.: *From axiom to dialogue: a philosophical study of logics and argumentations*, de Gruyter, Berlin, 1982.

GETHMANN, Carl Friedrich: *Protologik, Untersuchungen zur formalen Pragmatik von Begründungsdiskursen*, Suhrkamp Verlag, Frankfurt, 1979.

HAREL, David: *First-order dynamic logic*, Springer, Heidelberg, 1979.

HINTIKKA, Jaakko & HINTIKKA, Merrill B.: "Sherlock Holmes confronts modern logic : toward a theory of information--seeking through questioning", in: ECO, Umberto & SEBEOK, Thomas A. (eds.): *The sign of three, Dupin, Holmes, Peirce*, Indiana University Press, Bloomington, 1983, pp. 154—169.

HINTIKKA, Jaakko & KULAS, Jack: *The game of language, Studies in game-theoretical semantics and its applications*, Reidel, Dordrecht, 1983.

LINDSAY, Peter H. & NORMAN, Donald A.: *Human information processing*, Academic Press, New York, 1977.

VAN BENDEGEM, Jean Paul: "Pragmatics and mathematics, or how do mathematicians talk?", *Philosophica*, 29, 1982(1), pp. 97—118.

VAN BENDEGEM, Jean Paul: "The return of empirical mathematics", in: VANDAMME, Fernand (ed.): *Representation, use and acquisition of knowledge or the manufacture of knowledge*, Ass. Int. de Cybernétique, Namur, 1984, pp. 101—108.

WAHLSTER, Wolfgang: *Natürlichsprachliche Argumentation in Dialogsystemen*, Springer, Heidelberg, 1981.

WINSTON, Patrick Henry: *Artificial Intelligence*, Addison-Wesley; Reading, Massachusetts, 1977.