A SPECTRUM OF LOGICS OF QUESTIONING

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1. Interrogative games as illustrating the general problems of dialogue theory

This paper* is prompted by my belief that, before we try to develop a general theory of dialogues, it is advisable to try to gain first some real insights into some of the special kinds of dialogues which the prospective general theory eventually will have to cover.[1] The case study carried out in this paper concerns one particularly simple kind of dialogue, viz. question-answer sequences where all the questions are addressed by the same speaker, called the Inquirer, to the same answerer, called Nature.[2] The only important additional element is that the Inquirer is also allowed to draw logical inferences from Nature's answers plus an initial theoretical premise T. These simple dialogues were first devised as models of information-seeking (empirical inquiry). Here they are studied also as special cases of a potential general theory of dialogues, illustrating the richness of the phenomena such a theory will have to cover.

2. Interrogative games defined.

The main rules and other specifications of the question-answer dialogues to be studied here can be formulated by considering them as two-person games between the Inquirer and Nature. The following is a list of some of the most important specifications :

Players: There are two players, called the Inquirer and Nature.

Scorekeeping: The game is conducted by reference to two semantical *tableaux* in E. W. Beth's sense.[3]

Initial position: In both *tableaux* there is a theoretical premise T in the left column. One of the two *tableaux* has C in its right column, the other one ~C.

The aim of the game: The Inquirer is trying to close one of the two *tableaux;* Nature is trying to prevent closure. In other words, the Inquirer is trying to answer the principal or initial question "C or not-C?".

Moves: At each stage, the Inquirer has a choice between two different kinds of moves, a deductive move and an interrogative move. Each move is relative to the stage which some one *subtableau* has reached, and adds a formula to it.

Deductive moves: A deductive move is a step of *tableau* building in accordance to the usual rules of *tableau* construction. It is assumed that they do not contain any rules which can violate the subformula principle, such as the cut rule, *modus ponens,* etc.[4]

Interrogative moves: In an interrogative move, the Inquirer addresses a question to Nature. Nature's answer is added to the left column of the *subtableau* in question. A question naturally must not contain dummy names.

Different kinds of questions:[5] The Inquirer's question can be either a propositional question or a wh-question.

Propositional questions: A propositional question has the form "Is it the case that S_1, S_2, ..., or S_k?" An answer to it is one of the S_i (i = 1, 2, ..., k).

Among propositional questions, there are yes-or-no questions. They are of the form "S_o or not-S_o?".

Wh-questions: A wh-question is of the form "Which individual, call it x, is such that S[x]?"

Presuppositions: Before the Inquirer may ask a question, its presupposition must occur in the left column of the *subtableau* in question. The presupposition of a propositional question is

(1) $(S_1 \vee S_2 \vee \ldots \vee S_k)$.

The presupposition of a wh-question is

(2) (Ex)S[x].

For yes-or-no questions of the form "A or not-A?", where A is atomic, no presupposition is needed. (The presupposition would be in this case $(A \vee \sim A)$, i.e., a quantifier-free tautology.)

Language: The language used in the game is a finite first-order language L (without function symbols).

Model: Nature's being able to answer questions obviously presupposes that the game is played by reference to some one model M of L. It is normally (but not always) assumed that T is true in M.[6] I shall occasionally speak of worlds instead of models.

3. The importance of strategy considerations

The use of game-theoretical concepts in the definition of my interrogative model is not only an expositional convenience. It serves to call attention to what is probably the most important general methodological advantage of the model. In past studies of various kinds of dialogues, philosophers and linguists have typically formulated their concepts and theses in a way that, in terms of my model, apply to individual moves in the interrogative "game", corresponding to particular utterances in a dialogue or discourse. Examples are provided by speech-act theories,[7] whose very name betrays their conceptual focus; and Grice's conversational maxims.[8] There is a sense in which no such theory focusing on particular "moves" can be fully satisfactory, for from game theory we know that no values ("utilities") can in the last analysis be assigned to individual moves in the game, only to (complete) strategies.[9] In other words, there is no theoretically satisfactory way of relating particular moves to the general ends of the dialogue in question, in the case of my model, to the ends of inquiry. Thus theories of the kind just mentioned are bound to remain unsatisfactory in the last analysis. This fate is especially striking when it happens to a theory which, like Grice's, strives to base its theses on the aims of the kind of dialogue it is applicable to, so that the principles of conversational aptness become special cases of the general principles of rationality. I have elsewhere[10] shown how these limitations of Grice's enterprise are reflected by the details of his conversational maxims.

It is thus a major advantage of my interrogative model that is made possible and indeed focuses on questions of strategy and strategy selection. They will serve, I hope, as special cases of more general questions of dialogical (conversational) strategy. Such questions will be among the focal points of this survey. When I shall speak later in this paper of the "logic" of this or that questioning procedure, I mean in the first place the problems of optimal strategy selection.

4. The richness of the interrogative model

The details needed in the definition of my interrogative games are not likely to make much difference to one's initial impression of the simplicity of these questioning games. Basically, all that happens is that the Inquirer puts questions to Nature one by one and uses Nature's answers to them as additional premises for logical inferences. Can anything interesting be found by studying "dialogues" so utterly simple as such question-answer sequences ?

The answer is that my questioning games, far from being a simple or trivial objects of logical and semantical studies, in reality contain a tremendous wealth of interesting and important theoretical issues. Here I shall briefly identify some of the main logical and semantical perspectives opened by the interrogative model.

5. The fallacy of traditional "fallacies"

First, a partly historical point. My interrogative dialogues are obviously not unlike the questioning games played in Plato's Academy.[11] What is perhaps less obvious — or maybe just forgotten — is that Aristotle formulated his theory of so-called fallacies by reference to such dialectical games. Hence the customary discussions of the traditional fallacies which play such a major role in old-fashioned logic texts are to a considerable extent both historically inaccurate and systematically misconstrued.[12] Some of the central mistakes diagnosed by Aristotle are not at all fallacies in the modern sense of mistaken inferences, but rather violations of the rules of certain questioning games. By offering to us a framework for the systematic study of interrogative processes, my model makes it possible for the first time to put several of the traditional fallacies into a real theoretical perspective.

This can be spelled out in the case of individual "fallacies". For instance, the so-called fallacy of "begging the question", or *petitio principii*, is not a fallacy in our sense at all, for it does not deal with inferences in the first place. As Richard Robinson has shown,[13] it meant (in terms of the interrogative model defined above) for the Inquirer to ask ("petition") the big initial or principal question "C or not-C?" instead of trying to answer it by putting admissible "small" questions to Nature. There is no way of turning such a breach of the rules of my interrogative games to a mistaken inference.

The reason why *petitio principii* violates my game rules is that a question must not be asked by the Inquirer before its presupposition has been established. Of course, the presupposition of the main initial (principal) question is not available to the Inquirer in most cases — indeed, in all interesting cases.

Presuppositions of questions also play a role in other so-called fallacies. The most obvious one is the "fallacy" of many questions, exemplified by questions like

(3) When did you stop beating your wife ?

No argument is needed to show that we are not dealing with mistaken inferences here. What is involved in this "fallacy" is a violation of the requirement that the presupposition of a wh-question must be established before the question is asked. (Cf. section 2 above.)

Similar remarks can be made concerning other so-called fallacies. For instance, the *ignoratio elenchi* can be taken to be a mistake concerning the conditions on which my questioning procedure can be used to refute the initial theoretical premise T.

Thus the interrogative model puts a large part of traditional logico-semantical discussion to a new light by providing the first satisfactory framework for discussing Aristotelian "fallacies".

6. Questions and answers restricted

This is nevertheless only a small part of the story. The most important way in which the interrogative model can be seen to yield a veritable embarrassment of logical riches is to note that the description of the model given above is in certain respects incomplete. One of the things that have not yet been specified is under what conditions Nature will actually answer the Inquirer's questions.[14] It is obviously not necessary — nor desirable — to assume that Nature can answer everything that the Inquirer asks her. There are many kinds of possible restrictions, but two dimensions are especially important.

7. Nature not omniscient ?

First, Nature might not answer questions for reasons strcturally similar to ignorance. Heuristically, Nature may be considered in this case as not being omniscient. Nature will then give only such answers (conclusive answers) as are implied by what Nature knows (or, perhaps, as are in some suitable sense immediately implied by what Nature "knows"). When the interrogative model is used to discuss the scientific process and experiments and observations are conceptualized as questions to Nature such phenomena as limited observational accuracy fall under this heading. This dimension of possible restrictions on Nature's answers, and their implications for the Inquirer's questioning strategies, deserves closer scrutiny, which nevertheless will not be attempted here.[15]

8. EA-hierarchy of answers

Instead, I shall briefly discuss another dimension of possible restrictions. They are in terms of the logical complexity of the available answers. (The same restrictions can of course be formulated instead as pertaining to the logical form of the questions to which they are answers.) It can be assumed for simplicity that the answers are in the prenex form, i.e., have the form of a string of quantifiers followed by a quantifier-free expression. The number of changes of quantifier-kind (existential to universal or *vice versa*) will then be one important kind of measure of logical complexity.

More explicitly, I can define a hierarchy of prenexes as follows:

A quantifier-free expression has an $A^o = E^o$ prenex.

A string of existential quantifiers followed by an A^n-prenex is an E^{n+1}-prenex.

A string of universal quantifiers followed by an E^n-prenex is an A^{n+1}-prenex.

This hierarchy of prenexes defines a corresponding hierarchy of formulas and sentences they are prenexes to. I shall call it the EA-hierarchy.

9. A hierarchy of restraints on answers and its extreme cases

The types of restrictions which I shall consider in this paper are in terms of the prenexes of admissible answers in the EA-hierarchy just defined. The two extreme cases are the following :

(i) The unrestricted case. In this case, no restrictions are imposed on Nature's answers in terms of the EA-hierarchy. (Of course, this does not exclude other kinds of restrictions.)

(ii) The atomistic case. In this case, only quantifier-free answers are available. It is easily seen that this is to all practical purposes tantamount to allowing only yes-or-no questions concerning the truth or falsity of atomic sentences.

In order to gain some heuristic grasp of these two models, it may be useful to see what kinds of inquiries they can be applied to. The unrestricted case (i) can be thought of as being approximated by a clinical inquiry in a field where a great deal of advanced theoretical knowledge is available to the Inquirer, who is essentially applying it to a new case. Of course, the Inquirer does not have all the relevant knowledge in mind to begin with; it has to be brought to bear on the inquiry by means of suitable questions. Alternatively, the inquiry of a clever detective can be thought of as being of this kind. (Accordingly, I am tempted to call this the "Sherlock Holmes case".) A Sherlock Holmes or Hercule Poirot can appeal both to general truths about every day life ("the only person a trained watchdog doesn't bark at in the middle of the night is its master") and in some rare cases also to general scientific laws, over and above observations concerning particular cases.

10. The atomistic assumption

It is clear, however, that such ready availability of information about general laws is not characteristic of empirical inquiry in science. Mother Nature will not give direct answers to questions which deal with what happens always and everywhere or questions cocerning multiple interactions of all individuals in the world. Indeed, all questions an empirical scientist apparently can expect an answer deal with what happens in particular cases. What we can observe is what is happening here and now in *these* particular circumstances. Hence the atomistic assumption (ii) seems to be the best one to characterize empirical inquiry in science. There are some reasons to think that many recent as well as older philosophers of science have in effect been working on the basis of the atomistic assumption.[16]

11. A spectrum of logics of inquiry

One of the most remarkable features of the interrogative model is that the nature of the questioning process, including the choice of optimal questioning strategies, depends radically on the restrictions which may or may not be placed on available questions in terms of the EA-hierarchy. It might in fact be apter to speak of entirely different models of question-answer dialogues, except that the general framework into which I have placed these various models of inquiry also enables us to compare them with each other. The richness of logical and semantical problems which is generated by my approach is nowhere more dramatically in evidence than in the variety of logics of inquiry arising from different restriction in the EA-dimension.

This observation carries remarkable epistemological suggestions. What it shows is that there is no unique logic of inquiry. The nature of inquiry, including the search for optimal strategies of inquiry, depends essentially on the precise conditions in which the inquiry takes place. Whether this applies also to scientific inquiry remains to be discussed. (Cf. below, end of section 18.)

12. The logic of unrestricted interrogation

First, what is the logic of the "Sherlock Holmes" case, i.e., the logic of inquiry unrestricted in the EA-dimension? For simplicity, let's assume that there are no other restrictions present, either. Then at any stage of the *tableau* construction, there are as many questions the Inquirer can ask as there are (i) disjunctions and (ii) existentially quantified sentences (both without dummy names) in the left column of the *tableau*. For these are then all and only sentences which can serve as presuppositions of answerable questions.

In both cases, the same sentence can be used also as a basis of a *tableau* rule. However, asking the corresponding question is never disadvantageous and frequently advantageous to the Inquirer. For instance, consider the following situation :

(4)	T	C
	—	—
	—	—
	—	—
	—	—
	(Ex) S[x]	
	—	
	—	
	—	

Here the Inquirer can either (i) add to the left column a formula of the form S[α], where "α" is a new dummy name. Alternatively, the

Inquirer can use (Ex)S[x] for the purpose of asking the wh-question whose presupposition it is. The answer will be of the form S[b], where "b" is the name of some member of the domain of individuals do(M) of the model M.

Which outcome is better for the Inquirer ? Obviously the latter, for there normally is more that can be found out about the real individual b through further questioning than about the "arbitrarily chosen" dummy individual represented by "α".

Analogously, it is normally advantageous for the Inquirer to use a disjunction (S_1 v S_2) in the left column as a presupposition of a question rather than as a step in a purely deductive *tableau* construction.

These observations yield a preliminary and approximate but nevertheless important answer to the question: What is the logic of unrestricted ("Sherlock Holmes type") inquiry ? What my observations show that in this case questions (interrogative moves) can always (subject, of course, to other kinds of possible restrictions) replace deductive moves. This makes the overall questioning procedure, which is calculated to model empirical inquiry, structurally similar to the corresponding purely deductive procedure, assuming that the Inquirer is pursuing an optimal strategy. In this sense it can be said that *the logic of unrestricted question-answer dialogues is very close to ordinary deductive logic (proof theory).*

This point can be strengthened further. It is known from proof-theoretical studies that the crucial consideration in deductive strategy selection is the selection of an existentially quantified formula to be instantiated (from the ones that at the given stage of deduction are available for the purpose). But we just saw that these existential instantiations are precisely analogous to the selection of the wh-question to be asked at that stage of the inquiry. Hence the problem of optimal strategy choice is closely similar in the two cases.

The importance of this observation can perhaps be highlighted by recalling that the optimal deductive strategies have been shown not to be mechanical (recursive) in general. Even though I cannot at this time present an explicit formal proof, this result suggests, in virtue of the analogy between deduction and questioning noted earlier, that empirical discovery is not subject to mechanical rules, either, at least not in the unrestricted case.

Another corollary of the partial analogy is a vindication of the traditional idea that deduction, deductive logic and logical inference are valuable tools in substantial empirical inquiry. Even though I have not here cast any doubts on the idea that purely logical inferences are tautological, we have found that the problems of strategy selection are closely related in the purely deductive case and in the case of unrestricted interrogation. Even if this analogy should not be extendible to other kinds of questioning procedures, it serves forcefully to vindicate what I have called the Sherlock Holmes conception of logic and deduction.

13. The role of the subformula property

Even some of the disanalogies that there are between the two procedures, deduction and unrestricted interrogative inquiry, only serve to reinforce the strategic similarity between them. One feature which distinguishes unrestricted questioning procedures from deduction is that the logic of these questioning procedures does not satisfy the subformula property. In other words, no cut-elimination theorem is possible in the unrestricted case. In still other words, being allowed to adjoin tautological premises of the form $S_i \vee \sim S_i$ to the left column of the Inquirer's *tableau* can increase the range of what can be established by means of the interrogative process, depending of course on the selection of the admissible tautologies of this form. As an extreme case, if the tautologies in question are not restricted at all, we obtain a kind of completeness result for the unlimited case.

Completeness Theorem: By means of an unlimited questioning process, with arbitrary tautologies $S_i \vee \sim S_i$ adjoinable to the left column of the game *tableau*, conducted on a model M, any sentence S_o true in M can be established.

Completeness Theorem can easily be proved by induction on the number of logical symbols in S_o.

The reason why the failure of the subformula property does not detach the problem of strategy selection in unrestricted questioning from its counterpart on the deductive side is that the choice of suitable tautological premises $S_i \vee \sim S_i$ plays a crucial role in the search of optimal deductive strategies, too. Even though the same conclusions can on the deductive side be proved without such tautological premises, it is known that their use can shorten deductive proof essentially. Hence the choice of suitable tautologies to be used as additional premises plays an essential role in the search for optimal strategies both in pure deduction and in unlimited questioning, albeit in a different way.

14. The logic of the atomistic case: model-oriented logic

The atomistic case produces an entirely different kind of logic. Since I have studied some of its main features elsewhere,[17] suffice it to summarize the main facts.

The logic of atomistic questioning is basically the logic of what is logically implied not by the initial theoretical premise T alone, but by T together with the diagram (state-description) $\Delta(M)$ of the model M by reference to which the questioning procedure takes place. (The diagram $\Delta(M)$ is the set of all negated or unnegated atomic sentences true in M.) Although results concerning questions as to what can be done on the basis of $\{T\} \cup \Delta(M)$ are found in the literature, they have not been brought together and systematized. I have proposed to call a systematic study of the resulting logic *model-oriented logic*. Among the results that belong to model-oriented logic and illustrate its nature, there are the following :

(i) An optimal theory T in the atomistic case is not a complete theory, but a model-complete theory.

(ii) The counterpart to definability in model-oriented logic is identifiability (in the sense used in econometrics). Such identifiability is manifested in the form of definitions whose *definiens* depends on a finite number of members of do(M) (i.e., contains their names, if we are dealing with a particular model).

(ii) One appropriate tool for studying model-oriented logic are constituents and normal forms.[18] In model-oriented logic we study what happens at successively deeper-lying layers of constituents, whereas in "normal" logic (e.g., in the study of "normal" definability) we study what happens at the surface of constituents.

Not unexpectedly, the study of the atomistic case can also be brought to bear on a variety of important issues in the philosophy of science.

15. Atomistic case and explicit definitions

The properties of model-oriented logic would deserve further study. For instance, in this extreme case, the subformula property (cut elimination) does hold. However, another familiar logical principle goes by the board, viz. the noncreativity of explicit definitions. This result is not surprising in itself. The introduction of a new primitive predicate P by an explicit definition

(5) $(Ax)(Px \leftrightarrow D[x])$

has the effect of turning the complex predicate D[x] into a simple one and hence making questions concerning its applicability to particular cases answerable by Nature, even though they previously could not be answered by Nature in virtue of the atomistic assumption.

In spite of this naturalness of the result, it helps to shed interesting light on the nature of definitions and their role in scientific inquiry.

16. Revisiting the spectrum of interrogative logics

One of the most important features of the overall conceptual situation is nevertheless that the atomistic case with its "model-oriented logic" is only one extreme in a long spectrum of essentially different logics of questioning (inquiry). It is hence possible and indeed interesting to raise questions concerning the intermediate logics of questioning, for instance, the question as to in which of them the subformula property or the noncreativity of explicit definitions holds good. On the level of general philosophical issues, the existence of the spectrum suggests, even though it does not yet prove it, that philosophers' quest of *the* logic of inquiry or *the* logic of scientific discovery is a pursuit of a chimera. What the appropriate logic of an inquiry is depends (among other things) on its position in the EA-spectrum of restrictions on available answers to the Inquirer's questions.

17. The inductivist case

What substantiates this conjecture and makes the study of the spectrum of logics of questioning even more interesting is that some of the intermediate cases have a great deal of intrinsic interest. For instance, the classical inductivist conception of inquiry can be taken to amount to saying that Nature can also give to the Inquirer A^1-answers, over and above A^o-answers. For the gist of the inductivist position is that by examining a sufficient number of particular cases an Inquirer can establish whether an empirical generalization from such particular cases holds or not. The inductivist conception can perhaps then be criticized by trying to show that this A^1-restriction does not take us essentially further than the atomistic restriction.

18. A^2 logic as the logic of experimental inquiry

Most importantly, it can be argued that some of the intermediate cases are much more realistic models of experimental inquiry than either the unrestricted or the atomistic extreme case. For what is it that a scientist can find out by means of a controlled experiment? Not just information about particular cases. What a scientist is typically trying to do in such an experiment is to vary one variable characteristic of the experimental situation and by observing another variable to find out how it depends on the controlled variable. In other words, the answer that a scientist can receive from a controlled experiment, thought of as a question put to Nature, is a functional dependence between two or more variables. Now the expression of such a dependence is not an atomic sentence nor an A^1 or E^1 sentence. It has at least the logical complexity of an A^2 sentence. Hence the true logic of experimental science is not model oriented logic or deductive logic. It is A^2 logic, which therefore comes as close as anything in my spectrum to the true logic of scientific inquiry in advanced experimental sciences.

Now what is A^2 logic like? The only honest answer is that apparently nobody knows. There are relevant results in the logical literature, but there does not seem much of a comprehension of the overall situation comparable, e.g., with what is known (and understood) of the atomistic case.

It can nevertheless be seen that the logic of the A^2 case differs sharply from most philosophers' ideas of the scientific method, which therefore stand in need of critical scrutiny.

The main point is clear. Whereas no complex nested-quantifier conclusions can be derived by means of my interrogative processes (including deduction) from atomistic answers or A^1 answers, without essential reliance on an initial complex general theory T, such conclusions are possible at least in some situations for A^2 answers. This puts to a new light many things in the history and philosophy of science, among them Newton's claim of having "derived" or even "deduced" general laws from "phenomena". This claim, which most philosophers of science have been unable to make much sense of, now receives a perfectly natural interpretation. What Newton is saying is that general laws can be derived from answers to A^2 questions, i.e., derived by analysing phenomena by means of controlled experiments. This account fits in well both with Newton's emphasis on the use of analysis, i.e., the study of functional dependencies between different factors, and with his keen awareness of the nature and role of controlled experiments.

This is only one example of the power of A^2 logic to throw new light on different kinds of inquiry (different kinds of questioning processes). More study of the logical situation is nevertheless needed as a basis of such applications.

It is also in order to note that for non-experimental sciences, for instance, for purely observational sciences, different EA-restrictions and hence a different logic of inquiry may be appropriate, and likewise for purely clinical investigations within an applied science (e.g., medicine).

19. Inquiry and dialogue

Thus the apparent simplicity of my questioning model turns out to cover a tremendous variety of different structure and applications. Some of the applications sketched in this paper are more relevant to a scientist's dialectical interaction with nature than to dialogues between human speakers. The reason is that restrictions of replies to atomistic or A^1 or A^2 answers is more characteristic of a scientist's situation vis-à-vis nature than of what one is likely to face in a conversational situation in everyday life. But this is merely a matter of degree, as the example of Socratic dialogues vividly shows. (There is no reason why Socratic or Aristotelian questioning dialogues could not be restrained, e.g., by the atomistic assumption.) Indeed, Socratic *elenchus* is likely to be one of the test cases that first occur to a philosopher. Hence the rarity of such dialogues in the rough-and-tumble modern ordinary discourse does not reflect on the significance of the question-answer procedures codified in my interrogative games also as a case study designed to illustrate the richness with a general theory of dialogues will have to cover.

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NOTES

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[1] Along different lines, the same strategy has been followed by Lauri Carlson in his important book *Dialogue Games: An Approach to Discourse Analysis*, D. Reidel, Dordrecht, 1983.

[2] These question-answer dialogues and/or certain similar ones have been studied in my earlier papers, including the following : "The Logic of Science As a Model-Oriented Logic", in P. D. Asquith and P. Kitcher, editors, *PSA 1984*, Philosophy of Science Association, East Lansing, MI, 1984, pp. 177—185; "Rules, Utilities, and Strategies in Dialogical Games", in Lucia Vaina and Jaakko Hintikka, eds., *Cognitive Constraints on Communication*, D. Reidel, Dordrecht, 1984, pp. 277—294; (with Merrill B. Hintikka) "Sherlock Holmes Confronts Modern Logic", in E. M. Barth and J. L. Martens, eds., *Theory of Argumentation*, Benjamins, Amsterdam, 1983, pp. 55—76; "The Logic of Information-Seeking Dialogues: A Model" in W. Becker and W. Essler, eds., *Konzepte der Dialektik*, Vittorio Klostermann, Frankfurt am Main, 1981, pp. 212—231.

[3] For the *tableau* method, see E. W. Beth, "Semantic Entailment and Formal Derivability", Mededelingen van de Koninklijke Nederlandse Akademie van Wetenschappen, Afd. Letterkunde, N.R., vol. 18, no. 13, Amsterdam, 1955, pp. 309—342; reprinted in Jaakko Hintikka, ed., *Philosophy of Mathematics*, Oxford U.P., 1969, pp. 9—41.

[4] The usual basic rules always introduce subformulas of earlier ones. The others can all be reduced to the introduction of tautological premises of the form (S v ~S) into the left column, where the relation of S to earlier entries in the *tableau* is not restricted in any way.

[5] For a systematic discussion of different kinds of questions, of the question-answer relation, and of the concept of presupposition, see Jaakko Hintikka, *The Semantics of Questions and the Questions of Semantics*, Acta Philosophica Fennica, vol. 28, no. 4, Helsinki, 1976.

[6] When this assumption is relaxed, we can study, for instance, how T can perhaps be refuted by means of the questioning process.

[7] See, e.g., John Searle, *Speech Acts*, Cambridge U.P., 1969; Peter Cole and Jerry L. Morgan, eds., *Speech Acts (Syntax and Semantics*, vol. 3), Academic Press, New York, 1975.

[8] Paul Grice, "Logic and Conversation", in Cole and Morgan, note 7 above, pp. 41—58.

[9]The outcome of a game (the payoffs of different players) is not determined in general before the strategies of all players are determined. Conversely, the outcome is uniquely determined when each player has chosen a pure strategy, and the expected outcome is determined when each of the players has chosen a mixed strategy.

[10]Jaakko Hintikka, "Logic of Conversation as a Logic of Dialogue", in R. Grandy, editor, *Philosophical Grounds of Rationality: Intentions, Categories and Ends — A Festschrift for Paul Grice*, Oxford U.P., 1985, pp. 257—274.

[11]For a spirited description of these games, see Gilbert Ryle, "Dialectic in the Academy", in R. Bambrough, ed., *New Essays on Plato and Aristotle*, Routledge and Kegan Paul, London, 1965.

[12]For an excellent survey of the traditional fallacies, see C. L. Hamblin, *Fallacies*, Methuen, London, 1970.

[13]Richard Robinson, "Begging the Question 1971", *Analysis* vol. 31 (1971), pp. 113—117.

[14]Another thing that remains to be specified more closely is the determination of payoffs. For some remarks on the problem, see my papers, "Rules, Utilities, and Strategies in Dialogical Games" (note 2 above).

[15]The main upshot is to turn my questioning procedures into a quest of "interpolation formulas" in logicians' sense. This does not disassociate the procedures from deductive logic in the least.

[16]For instance, both inductivist and hypothetico-deductive theories of science usually assume that a theory can be confronted with experience only in terms of particular consequences of the theory.

[17]See "The Logic of Science as a Model-Oriented Logic", note 2 above.

[18]For them, see, e.g., Veikko Rantala, *Aspects of Definability*, Acta Philosophica Fennica, vol. 29, nos. 2—3, Helsinki, 1977.