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FORMAL THEORY OF GROUP ACTIONS AND ITS APPLICATIONS

Maria Nowakowska

1 *Intruduction*

The idea underlying the construction of the theory is that composite actions can be represented as strings of elementary actions, and the latter cannot, in general, be performed in arbitrary order. This observation alone suffices to draw linguistic analogy : there must be a class of sensible orders in which the actions can be performed, in the same way as there is a class of sensible orderings of letters under which they form words — as opposed to nonsensical strings, and also — a class of sensible orderings of words into sentences — as opposed to syntactically incorrect utterances.

This analogy has numerous fruitful consequences, since it allows us to use the concepts of formal linguistics for the study of actions. One could call this study "formal grammar of actions".

Moreover, when uttering sentences, one usually wants to express some meaning. In similar way, the actions, whether composite or elementary, are performed in order to attain some goals, simple or composite, that is, to cause the occurrence of some events at specific times.

This is, in most compact way, the essence of basic ideas of formal theory of actions, as presented in "Language of Motivation and Language of Actions", and developed in further works (see Bibliography).

In other formulation, one may also say that actions are treated as "words" of a certain vocabulary, and admissible strings of actions — as expressions in the "language of actions". Furthermore, outcomes of strings of actions are regarded as "meanings", hence constitute the semantics of the so defined language of actions.

The extension to the case of actions of more than one person, which is the main object of this paper, leads to considering parallel strings of actions, each performed by one person (to be called k-strings). Though there is no direct linguistic analogy with the natural language, one can still treat such bunches of parallel strings of actions by means of mathematical linguistics, and speak of "multidimensional language of actions".

The suggested theory of actions has interdisciplinary character; it integrates within one system the psychological, linguistic, logical and decisional aspects of behaviour. In short, one may say that formal theory of actions is a system which unifies thought and action. Not all these aspects will be covered in this paper, of course. However, the next section will contain an outline of the results obtained in various possible directions of research. Subsequent sections will cover a presentation and development of the theory for the case of group actions, and then, an application will be shown, concerning communication theory.

2. *An outline of research directions and results*

Formal analysis of the system may be carried out quite independently of the different possible interpretations, though, of course, it is guided by them. The main lines of research are :

1) Analysis of the structure of language of actions, through an application of some concepts borrowed directly from formal linguistics, such as generative grammars, distributional classes, parasitic strings, etc. These concepts acquire then action-theoretical interpretation, and become useful tools in explicating the "grammatical" features of actional situations. This line of research will be illustrated in subsequent sections.

2) Analysis of attainability of some outcomes, or more generally, configurations of outcomes, that is, composite goals. If a given goal is attainable, that is, there exists at least one string (k-string) of actions leading to it, than the natural topic of study are various types of efficiency and optimality. Here one gets a possibility of explication of some praxiological concepts, such as decisive moments, complete possibility, etc. These topics will also be briefly discussed in the next sections.

If a given goal is not attainable, one obtains a model of conflict of interests, or motives, since in this case one is forced to make some resignation from certain goals in order to attain others.

A rather unexpected result here is that a relatively simple conflict of four competing motives which cannot — say, because of situational constraints — be satisfied jointly, can take on one of the 113 non-equivalent forms, depending on which of the pairs or triplets of motives are jointly satisfiable and which are not.

This taxonomy of internal conflicts (see Nowakowska 1973), encompasses in particular conflicts introduced in dynamic psychology (such as "approach-approach", "approach-avoidance" etc. as well as conflicts between Id and Superego).

3) The third line of research consists of studying the relations between verbal behaviour, that is, utterances, and non-verbal behaviour. Here one can distinguish in the "vocabulary of actions" a subset, consisting of actions having the form of utterances. By restricting these utterances to those in which a person evaluates, explains, justifies, etc. his past or future actions, one obtains a fragment of natural language, which constitutes a linguistic representation of motivation.

This line of research may be subdivided as follows :

3a) a decision model based on the assumption that choice is determined by evaluation of alternatives on various scales of "motivational space". An example of such a model is the SEU model, which combines utilities and subjective probabilities. Generally, even if one leaves the nature of these scales and decision function unspecified, one can apply the theorem of Arrow ("Social Choice and Individual Values", 1963) by treating the decision as "social choice" and particular evaluations as "votes". In other words, one can conceptualize the internal decision as resulting from "inner voting", hence as a form of social choice. One obtains in this way certain very general deductive consequences concerning the rules of decision making, corresponding to some ways of solving the conflicts known in dynamic psychology.

3b) Secondly, one can develop here a sort of "motivational calculus", by studying the rules of inference from those utterances in natural language, which involve motivational functors, such as "I want", "I ought to", "I prefer", "I am glad that", "I know", and so on. These rules allow, in turn, to formalize the inference on motivation, by using the principles analogous to those known in logical theory of enthymemes and sorites (i.e. tacit premises).

3c) Finally, one may explore the structural aspects of the behaviour which is consistent, or inconsistent, with certain motivational utterances, for instance those which assert a promise. Here one can prove that under some rather general conditions, the set of strings which satisfy a promise is a context free language, in the sense of Chomsky (1963). This result characterizes the general regularities of the structure of social norms.

Generally, an analysis of motivational consistency of verbal and non-verbal behaviour leads to a formal theory of planning behaviour, where elaboration of the plan of actions is identified with generation of a sentence (that is : a string of actions) in generative grammars.

4) The fourth line of research is connected with the ethical valuations of strings of actions leading to ethically diverse outcomes. A hypothesis here is that there are two principal strategies of such valuations, from which all others can be derived. These strategies may be named "puristic", where one takes into account only that outcome which is ethically worst; the other strategy may be called "liberal", and it involves some sort of "averaging", or taking into account "mitigating circumstances". Whether the second strategy can be used meaningfully, depends on conditions for the existence of the so-called conjoint measurement in particular context; whether such a strategy is actually applied depends, of course, on the personality of the judge.

5) The last line of the theoretical development of the formal system, which is the main object of this paper, is an extension to cover the case of joint actions of many persons, both cooperating and conflicting.

Each of these five directions of research offers numerous possibilities of further extensions, beyond the present status of development of the theory. In addition to that, possibilities of new applications, even within the present conceptual framework, are obtained by varying the interpretation of the concept of "admissibility" of the string of actions, i.e. the concept of "language of actions". Here the main interpretations, and their consequences, are as follows :

A. Language of actions may be interpreted as the class of all strings of actions which are physically possible to perform. This is the most natural interpretation, and it leads to various structural analyses of specific action situations, in particular those studied in organization of work, and psychology of work.

B. Secondly, language of actions (of one person) may consist of all those strings of actions which are psychologically admissible for this person. The class of all such "languages of actions" may constitute a basis for creating a special personality theory, in which the concept of personality would be explicated and taxonomized in terms of "grammatical rules" for various languages of actions.

C. If one considers social admissibility, that is, consistency with a given social role, the analysis might lead to structural characterization of social roles, and to their taxonomy.

D. Next, one can take under consideration the organizational admissibility, that is, acceptability in a given organization, according to the rules, customs, norms, etc. of this organization. This analysis leads to foundations of the theory of organizations.

E. One can also consider admissibility from the point of view of a given theory or methodology. This analysis leads to explication of various logical relations between theories, methodologies, etc.

F. Next, one can consider admissibility from the point of view of a given object. The idea here is that each object generates a set of appropriate strings of actions, admissible in view of the structure of this object. In other words, one can speak of actions which are "grammatical" for a given object, and those which are not grammatical (as an example, one can think of objects such as computers, and the actions of operating them).

In case of a certain configuration of objects related in some way among themselves (by relations such as "being a part of", "domination", etc.), there arises a problem of relations between the "languages" of these objects, possibly expressible in terms of set-theoretical operations. It might seem that the configurations of larger number of objects, with complicated relational structures, would impose more constraints on the generated actions, than the "looser" configurations, with less number of objects.Easy examples, however, show that this need not be true in general.

A particular case of configuration of objects is the environment; it generates therefore a set of appropriate actions. All other actions, from outside this set, are therefore diagnostic in the sense that they represent those aspects of activity which are related to internal factors of acting person, his motivation, plan of actions, etc.

Analogously, problem is also a particular configuration of objects (stimuli), and generates its set of actions — solutions (possibly this set is empty, or consists of one element only). Thus, one can consider "grammars" of problems, that is, algorithms of solving them.

G. FInally, another possible interpretation of the concept of admissibility, to be discussed in some detail in this paper, is connected with communication theory, when one considers communication carried out by several media simultaneously (e.g. verbal medium, medium of gestures, body movements, etc.). The acting "persons" here correspond to different media, each producing a string of actions.

3. *Formal system of actions of many persons*

The main features of the formal system of actions outlined below is that it allows the conceptual separation of three components :

a) Those relating to the possibility of actions, regardless of the goals for which these actions are to be undertaken;

b) those relating to outcomes of actions, again regardless of the goals, i.e. of particular system of preferences of acting persons among outcomes;

c) those relating to the mutual interrelations of goals of actions of particular persons.

Accordingly, the system takes on the form of eight primitive notions

(1) $\langle K, D, \# , \varphi , L , S, R, \pi \rangle$

of which the first five concern the structural properties of actions, the next two (S and R) — outcomes of actions and the last one (π) — the goals for particular persons

These notions will now be interpreted and discussed successively, together with auxiliary definitions and theorems.

4. *Structure of actions*

An already stated, the first five concepts in system (1) concern the structural properties of actions, without reference to their outcomes or goals. The interpretation of these concepts is as follows.

Acting persons. The first primitive concept, K, will be interpreted as the set of all acting persons considered in the given situation. In the sequal, k will stand for the number of elements of K.

It ought to be mentioned here that K need not form a team, i.e. set of persons characterized by a common goal : K may equally well be decomposable into two or more opposing or competing subgroups of persons.

Elementary actions. The second primitive concept, D, is the set of actions which will be called elementary. As a primitive concept, the meaning of the term "elementary" cannot be defined formally; the intuitive idea is to choose, in any particular situation to be described, the elements of D in such a way that every other action of interest can be represented as a string of elements of D.

Idling. The symbol # will denote a distinguished element of D, to be referred to as "idling" (non-action). In practical applications, # does not have to signify refraining from any activity; it may denote any action different from the ones which are of main interest in the considered context (so that in effect, # is rather a class of actions, all considered to be equivalent).

Duration. The fourth primitive concept, φ , is a function defined on D; the value φ (d) is interpreted as the number of units of time necessary to perform the action d. It is assumed that function φ is integer-valued; in practice, it means that the time unit is chosen in such a way that every action takes a certain integer number of units of time to be performed.

Moreover, it will be assumed that

(2) $\varphi(\#) = 1;$

longer periods of idling will therefore be denoted by concatenations of the form $\#\#\# \quad \ldots$.

Before explaining the fifth primitive concept, L, it is necessary to introduce some auxiliary notions.

Let

(3) $D^* = \{ d_1 d_2 \ldots d_n : d_i \in D,\ i = 1, \ldots, n.\ n = \ 0,1,2,\ldots \}$

be the class of all strings of actions from D (with possible repetitions), of any length. We include into D* the empty string, with n = 0.

The function φ may be extended from D to D* as follows : if $u = d_1 d_2 \ldots d_n$ is a string of actions in D*, then its duration equals

(4) $\varphi(u) = \varphi(d_1) + \varphi(d_2) + \ldots + \varphi(d_n).$

It is important to note that D* contains all strings of actions, regardless whether they are physically possible to perform or not.

Next, let

(5) $D_k^* = \{ (u_1,\ldots,u_k) : u_i \in D^*,\ i=1,\ldots,k;\ \varphi(u_1) = \ldots = \varphi(u_k) \}.$

Thus, $\bar{D}_k^*$ denotes that part of the Cartesian product D* x ... xD* (k times) which consists of k—tuples $u = (u_1,\ldots,u_k)$ of strings of actions with the same duration.

The elements of D_k^* will be interpreted as *simultaneous* strings of actions from D performed by members of K (to be called simply *k-strings*): the first member of K performs the string u_1, the second performs u_2, and so on. Again, D_k^* contains all k-strings, regardless whether they are physically possible to perform or not.

Language of actions. The fifth primitive concept, L, is a subset of D_k^*, interpreted as the set of all those k-strings which are possible to perform. Here the notion of possibility may be interpreted in several different ways The most important of them is the physical possibility; another one may be psychological possibility, or consistency with a given social role.

Before describing further primitive concepts, it may be worth while to illustrate briefly how the above five concepts may be used to introduce a certain taxonomy of strings of actions, and define the notions of cooperation and blocking. It will also be shown how certain notions taken from formal linguistics can be applied to describe the structure of the class L.

5. *Taxonomy of strings of actions*

Firstly, for $i = 1,\dots,k$ define

(6) $L_i = \{ u_i \in D^* : (u_1,\dots,u_i\dots,u_k) \in L$ for some $u_1\dots,u_{i-1},u_{i+1},\dots,u_k \in D^* \}$

and

(7) $D_i = \{ d \in D: w_1 d w_2 \in L_i$ for some $w_1,w_2 \in D^*\}$.

Clearly, L_i (language of actions of person i) is the set of those strings of actions which person i can perform in some circumstances (i.e. when other persons perform some suitable strings of actions), while D_i is the set of elementary actions appearing in L_i. Thus, it is appropriate to refer to D_i as to the *repertoire of actions of person i*.

We may reasonably assume the conditions :

(8) $\# \in D_i$ for all i,

and

(9) $\bigcup_{i=1}^{k} D_i = D$.

The first of these conditions asserts that each person is capable of idling, while condition (9) means that every action in D is in the repertoire of at least one member of the set K.

For the taxonomy of strings of actions, let us restrict the considerations to the case $k = 2$ (two acting persons).

The following definitions cover the most important cases which may occur.

Definition 1. Sets $A \subset L_1$ and $B \subset L_2$ are *independent*, if $(u,v) \in L$ for all $u \in A$. $v \in B$.

This means that whenever person 2 performs any string of actions from the set B, person 1 can perform any string of actions from the set A (and vice versa).

Obviously, if $(u,v) \in L$, then the sets $A = \{u\}$ and $B = \{v\}$ each consisting of one element only are independent. We have also the following

Proposition 1. If $A_1, A_2 \subset L_1$ are both independent of the set $B \subset L_2$, then $A_1 \cup A_2$ is also independent of B. If $A \subset L_1$ and $B \subset L_2$ are independent, then any subset $C \subset A$ is also independent of B.

An analogous proposition is also true for subsets of L_2.

Thus, unions are subsets of sets in L_1 (resp. L_2), independent from a fixed set in L_2 (resp. L_1) are again independent.

For a fixed set $B \subset L_2$ we may therefore introduce the concept of *maximal* subset $A_B \subset L_1$ which is independent of B. Formally, this maximal set A_B and B are independent, and for any $u \in L_1$ A_B there is a $v \in B$ such that $(u,v) \notin L$.

In particular case when $B = \{v\}$, the maximal set A_B equals simply the set of strings of actions of person 1, which he can perform if person 2 performs the string v.

We shall now introduce

Definition 2 The string $u \in L_1$ is *indispensable* for the string $v \in L_2$ if $(u,v) \in L$ and$(u',v) \notin L$ for any $u' \in L_1$ with $u \neq u'$.

This means that person 2 cannot perform v unless person 1 performs u.

In a similar way we define the concept of indispensability of $v \in L_2$ for $u \in L_1$.

We have therefore

Proposition 2. Let $u \in L_1$ be indispensable for $v \in L_2$. If $A \subset L_1$ and $B \subset L_2$ are independent,and A contains any string different from u (i.e. $A \neq \{u\}$), then $v \notin B$.

Indeed, suppose that assertion is not true, i.e. A and B are independent, $v \in B$ and there exists $u' \neq u$ with $u' \in A$ Then $(u',v) \in L$ by definition of independence, contrary to the assumption that u is indispensable for v.

Definition 3. Strings of actions $u \in L_1$ and $v \in L_2$ are *strictly cooperative* if each of them is indispensable for the other.

This means that $(u,v) \in L$, but no pair of the form (u',v) or (u,v') with $u \neq u'$ and $v \neq v'$ is in L.

Let us now turn to the definition of blocking.

Definition 4. A string of actions $u \in L_1$ is ***blocking*** for the string v $\in L_2$, if $(u, \# \varphi(u)) \in L$ and $(u,v) \notin L$. Here $\# \varphi(u)$ means idling for the duration of time $\varphi(u)$.

Thus, u is blocking for v, if person 1 is free to perform u (i.e. he can perform it when the person 2 is idling), while performing u is impossible together with the string v.

The above concepts can be carried down from the level of strings of actions to the level of elementary actions, thus leading to a taxonomy of elements of D.

On the other hand, in case of more than two acting persons, the situation becomes much more complicated, except for the case when the set K can be in a natural way partitioned into two subgroups of persons. If one of these subgroups consists of r persons, and the other — of m = k — r persons, one can consider separately the r-strings and the m-strings, performed simultaneously by members of the two subgroups, and define the above notions (of independence, indispensibility, cooperation etc) as above, replacing strings u and v by r-strings and m-strings.

6. *Parasitic strings and equivalence classes*

The main instrument for analysing the structure of languages of actions L_i and L are the concepts of parasitic strings and of A-equivalence.

We shall present the definitions of these concepts for the case of one-person language of actions L_i; the generalizations for the case of k-strings is immediate.

Definition 5. A string of actions $u \in D^*$ is L_i-*parasitic*, if $w_1uw_2 \notin L_i$ for any $w_1, w_2 \in D^*$.

In other words, u is L_i-parasitic, if it cannot be a part of any string of actions in L_i.

The most interesting case is, of course, when some strings of actions are parasitic with respect to some L_i, and not parasitic with respect to some other L_j. A string u with this property is such that person i cannot perform it, while person j can; this naturally leads to defining the concept of qualifications (or : competence) of persons from set K, hence to a differentiation of this set.

As examples here, one may consider k-strings of actions of such teams as, say, surgery team performing an operation, or an airplane crew. The "grammars" of actions in such teams are reasonably well defined, that is, the classes of parasitic strings are rather rich. Also, the concepts of repertoire of actions and of qualifications acquire an obvious significance for the above examples.

Let us now introduce

Definition 6. For a given set $A \subset D^*$, let $u \sim_A v$ if for every w_1, $w_2 \in D^*$ the conditions $w_1uw_2 \in A$ and $w_1 vw_2 \in A$ are equivalent.

It is easy to prove that for any A the relation $\sim_A$ is an equivalence among all strings of actions; thus, the set of strings of actions will split into equivalence classes. Moreover, we have

Proposition 3. For $A = L_i$, all parasitic strings are $\sim_{L_i}$-equivalent.

Considering partitions into equivalence classes of relations $\sim_A$ for various sets A, especially for those sets which are defined by means of outcomes (see next section), one can define the concepts such as actions *crucial* for a given goal, etc. For the details of construction and proofs of theorems, see Nowakowska 1973, Chapter IV.

7. *Algebra of goals and algebra of means*

We shall now characterize the two primitive concepts of system (1) relating to outcomes of actions, namely S and R.

Thus, S will be a set, whose elements will be called *outcomes*, or *results* of actions.

No temporal aspect need to be stated explicitly here : should one want to consider them in any particular application, all it suffices to do is to interpret elements of S as occurrences of some events at given moments of time (that is, "event E at time t_1" and "event E at time t_2" are considered as distinct elements of S, if only $t_1 \neq t_2$).

Finally, R will be a binary relation in L x S; the symbol uRs, where $u \in L$ and $s \in S$ means that performing the k-string u leads (among others) to the outcome s.

Of course, one k-string may lead to different outcomes, and one outcome may be brought about by different k-strings.

Generally, let

$$(10)\ R^{-1}(s) = \{u : uRs\}$$

be the set of all k-strings which lead to the outcome s. Further, for $Q \subset S$, let

$$(11)\ R^{-1}(Q) = \bigcup_{s \in Q} R^{-1}(s)$$

$$(12)\ \hat{R}^{-1}(Q) = \bigcap_{s \in S} R^{-1}(s).$$

Thus, $R^{-1}(Q)$ is the set of all k-strings which lead to *at least one* outcome from the set Q, while $\hat{R}^{-1}(Q)$ is the set of all k-strings which lead to *all* outcomes in the set Q.

The above formulas lead to establishing a certain isomorphism between goals and sets of k-strings of actions which lead to these goals, Generally, a goal may be described by combining elements of S with functors of conjunction, alternative and negation. For instance,

$$(13)\ (s_1 \ \&\ s_2) \vee (s_1 \ \&\ s_3 \ \&\ s_4) \vee (\sim s_3 \ \&\ \sim s_5)$$

is the goal : "either outcomes s_1 and s_2, or outcomes s_1, s_3 and s_4, or avoiding both outcomes s_3 and s_5". Thus, if f is any propositional function of n variables which involves only conjunction, alternative and negation, and $\{s_1,...,s_n\}$ is any set of outcomes, then $f(s_1,...,s_n)$ represents a certain goal expressible in terms of $s_1,...,s_n$.

The isomorphism in question, is as follows : the set of all k-strings of actions which lead to a composite goal $f(s_1,...,s_n)$ may be obtained by :

a) replacing each symbol s_i by the symbol $R^{-1}(s_i)$;

b) replacing each functor of conjunction by the set-theoretical operation of intersection;

c) replacing each symbol of alternative by the set-theoretical operation of union;

d) replacing each symbol of negation by the set-theoretical operation of complementation.

For instance, for the goal (13), the set of all k-strings of actions leading to this goal will be :

(14) $(R^{-1}(s_1) \cap R^{-1}(s_2)) \cup (R^{-1}(s_1) \cap R^{-1}(s_3) \cap R^{-1}(s_4)) \cup (L \setminus R^{-1}(s_3)) \cap (L \setminus R^{-1}(s_5))$;

using formulas (11) and (12) and de Morgan rule, the last formula may be written as

(15) $\hat{R}^{-1}(Q_1) \cup \hat{R}^{-1}(Q_2) \cup (L \setminus R^{-1}(Q_3))$.

where $Q_1 = \{s_1,s_2\}$, $Q_2 = \{s_1,s_3, s_4\}$ and $Q_3 = \{s_3,s_5\}$.

Such an algebraic representation allows the definition of various types of attainability. Thus, a set $Q \subset S$ is weakly or strongly attainable, depending on whether the sets $R^{-1}(Q)$ or $\hat{R}^{-1}(Q)$ are nonempty. The highest form of attainability is that of complete possibility, defined as follows.

Let Q be a subset of S. If $A \subset Q$, let $\tilde{A}$ denote the conjunction of all negations of outcomes in $Q \setminus A$.

Definition 7. We say that Q admits *complete possibility*. if for any $A \subset Q$ the set

(16) $\hat{R}^{-1}(A) \cap \hat{R}^{-1}(\tilde{A})$

is nonempty.

Thus, complete possibility means that for any set A of outcomes in Q it is possible to attain all outcomes in A, and avoid all outcomes in $Q \setminus A$.

Now, the possibilities of attaining or avoiding some outcomes can only diminish in time, i.e. be lost as a result of performing certain k-strings, which either eliminate some outcomes, or determine their occurrence (so that it is impossible to avoid them).

Formally, if $z = (z_1,\dots,z_k)$ is a k-string of actions, and if for every $A \subset Q$ the set (16) contains a k-string with the beginning z (i.e. a k-string of the form $(z_1 w_1,\dots,z_k w_k)$), then performing the k-string z does not destroy the complete possibility with respect to Q. The longest among all k-strings z with this property is the length of time during which one can avoid making any decision about the outcomes in Q, both positive ones and negative ones.

The concept of complete possibility is of some importance in organization and planning of actions.

8. *Praxiological sets*

In the sequel, let $\bar{S}$ denote the set of all goals as defined above, i.e. expressions built up of elements of S by means of functors of conjunction, alternative and negation.

Let G be any fixed goal in $\bar{S}$, and let $R^{-1}(G)$ be the set of all k-strings which lead to the goal G.

We shall introduce the following

Definition 8. Let $u,v \in R^{-1}(G)$. We say that the k-string $u = (u_1,\dots,u_k)$ is a *praxiological improvement* (for goal G) of the k-string $v = (v_1,\dots,v_k)$ if u can be obtained from v by replacing some of its actions by the action #.

In other words, u is a praxiological improvement of v (for goal G), if u is more "thrifty" in actions than v, and yet, leads to the same goal G.

Definition 9. If $v \in R^{-1}(G)$ and there is no k-string u which is a praxiological improvement of v, then the k-string v will be called *praxiological* for G. The set of all k-strings which are praxiological for G will be denoted by Prax G.

We have the following

Proposition 4. Prax $G \subset R^{-1}(G)$, and if the right hand side set is nonempty, so is the left hand side set.

The inclusion follows directly from the definition. The nonemptiness follows from the fact that if $u \in R^{-1}(G)$ and u is not in Prax G, the u can be reduced by replacing some of its actions by the action #. If the new string so obtained is still not in Prax G, it must be possible to reduce it further. However, such successive reductions cannot proceed indefinitely in view of the finiteness of the number of actions in each k-string : eventually one would obtain the k-string consisting of actions # only, which cannot be reduced any further.

The set Prax G consists of all k-strings which lead to the goal G and are, in some sense, most economical (i.e. contain only those actions which are necessary for G).

It may happen that the set Prax G contains some k-strings with the property that one or more persons is idling, i.e. performing the string of actions ###....

To each k-string $(u_1,\ldots,u_k)$ in Prax G assign the set of those persons j for which u_j is *not* of the form ## ... . This is, therefore, the set of all persons actively participating in attaining the goal G (in the k-string $u = (u_1,\ldots,u_k)$). Let $\mathbf{K}$ be the class of all subsets of K obtained in this way, for varying strings in Prax G.

If $\mathbf{K}$ contains only the set K, all persons are needed to attain G; otherwise it is possible to attain G with a smaller number of persons. One can therefore consider the class $\mathbf{K}^*$ of all *minimal* subsets of elements of $\mathbf{K}$, that is, the class $\mathbf{K}^*$ defined by the conditions :

$A \in \mathbf{K}^*$ if $A \in \mathbf{K}$ and the relations $B \subset A$, $B \neq A$ imply $B \in \mathbf{K}$.

The union $\bigcup_{A \in \mathbf{K}^*} A$ consists of all persons whose active contribution is needed in at least one praxiological way of attaining the goal G. The intersection $\bigcap_{A \in \mathbf{K}^*} A$ (which may be empty) consists of all those persons whose active contribution is needed in every praxiological k-string of actions leading to G. Finally, the complement $K \setminus \bigcup_{A \in \mathbf{K}^*} A$ consists of all persons whose active contribution is attaining the goal G can always be dispensed with.

These concepts are of obvious considerable importance for the organization and planning of actions.

9. *Preference system*

So far the concepts referred only to the possibilities of performing certain combinations of actions, and attaining certain combinations of outcomes. Within this framework it was not possible to express the attitude of particular persons from K towards the goals in $\bar{S}$.

The last primitive notion of system (1), denoted by π, is a system

$$(17)\quad \pi = \{ \geqslant_j, j \in K \}$$

where each $\geqslant_j$ is a binary relation on $\bar{S}$, satisfying the following conditions :

transitivity : for every $j \in K$ and $s,t,u \in \bar{S}$, if $s \geqslant_j t$ and $t \geqslant_j u$, then $s \geqslant_j u$;

reflexivity : for every $j \in K$ and $s \in \bar{S}$ we have $s \geqslant_j s$;

connectedness : for every $j \in K$ and $s,t, \in \bar{S}$ either $s \geqslant_j t$ or $t \geqslant_j s$ (or both).

We shall refer to $\geqslant_j$ as to the preference relation for the person j, and the symbol $s \geqslant_j t$ will be interpreted as (weak) preference of s to t by person j.

Generally, the concept of preference relations $\geqslant_j$ allow us to characterize those subsets of K which constitute teams, i.e. roughly speaking, groups of persons with identical preferences.

Let $\bar{S}_0$ be a distinguished subset of $\bar{S}$; intuitively $\bar{S}_0$ will consist of those goals which are important in the given considerations. We assume that $\bar{S}_0$ contains at least two distinct elements.

We shall now introduce

Definition 10. Two persons, $i,j, \in K$, are said to be $\bar{S}_0$*-equivalent*, if for every $x,y \in \bar{S}_0$ the condition $x \geqslant_j y$ holds if, and only if, $x \geqslant_j y$.

It is easy to show that the relation defined above is indeed an equivalence, i.e. it is reflexive, symmetric and transitive; consequently, the set K will split into $\bar{S}_0$-equivalence classes. Members of the same class will have identical preferences within the set of goals S_O, while members of different classes will have different preferences.

Any such $\bar{S}_0$-equivalence class may be called $\bar{S}_0$*-team*, and for any $\bar{S}_0 \subset \bar{S}$ the set K may consist of one or more $\bar{S}_0$-teams. It may also happen that each $\bar{S}_0$-equivalence class consists of one person only, in which case every member of K forms a separate "team").

For a description of a given situation one may now use the concepts of game theory, identifying strings of actions of one person as his "strategies". Of particular significance here are the concepts of equilibrium, and of coalition of players.

Definition 11. A k-string $(u_1^*,\dots,u_k^*) \in L$ is an *equilibrium* (of the action situation described by system (1)), if for all $i \in K$ and all u_i such that $(u_1^*,\dots,u_{i-1}^*, u_i, u_{i+1}^*,\dots,u_k^*) \in L$ we have

$$(18)\quad Q(u_1^*,\dots,u_k^*) \geqslant_i Q(u_1^*,\dots,u_{i-1}^*,u_i,u_{i+1}^*,\dots,u_k^*),$$

where $Q(u_1^*,\dots,u_k^*)$ is the conjunction of all outcomes in S which occur when the k-string $(u_1^*,\dots,u_k^*)$ is performed (that is, it equals to the conjunction of all outcomes s such that $(u_1^*,\dots,u_k^*)Rs$; the right hand side of (18) is interpreted in a similar way).

Intuitively, a k-string is an equilibrium, if for every person $i \in K$ the following is true : if he knows that other persons will perform strings of actions $u_1^*,u_2^*,\dots$ he can only lose (i.e. obtain something less preferred) if he performs any other string than u_i^*.

Equilibrium in a given situation described by system (1) need not be unique. To see this, put $\bar{S}_O = \bar{S}$ in Definition 10, and consider S-teams (i.e. groups of persons who have preferences identical in the whole set of goals). We have then

Proposition 5. Assume that for every $i,j \in K$, persons i and j are $\bar{S}$-equivalent (i.e. K forms an S-team), and let s* be the (common) most preferred goal (that is, $s^* \geqslant_j s$ for all $j \in K$ and $s \in \bar{S}$). Then any k-string u such that uRs is an equilibrium.

10. *Communication as a multidimensional language of actions*

The following sections will contain an application of the system of group actions to communication theory, or, more precisely, to construction of a system which incorporates both verbal and non-verbal communication. In this application, it will be necessary to modify slightly system (1), especially in its part regarding the semantics.

Conceptually, communication may be regarded as carried by several media, such as utterances, pitch of voice, speech disruptions, gestures, facial expressions, body movements, etc. For any application it will, of course, be necessary to specify the media exactly. However, for the formal theory it is not necessary.

When one considers communication carried by several media, the situation is very much analogous to team actions : every medium may be regarded as one member of the team, and all of them perform some actions, specific for a given medium. A combination of simultaneous performances of "elementary actions" on each medium forms an "elementary unit" of communication action, which will simply be called "unit". Then one can consider the "language of communication actions", identified with the class of all admissible strings of such units.

10.1. *The basic conceptual scheme*

The considered system will be based on the following sixtuplet of primitive concepts :

$$\langle M, V, \#, L, S, f \rangle$$

where M is the set of media, V is the vocabulary of actions, # is a distinguished action called "pause", L is the set of all admissible strings of units, called language of communication, S is the set of all meanings, and f is the function describing fuzzy semantics.

The concept of unit of communication

A communication unit, or simply a unit, will be defined as a function $h : M \to V$, i.e. as a function which assigns an action to each medium. If $M = \{m_0, m_1, \ldots, m_r\}$ is the set of all media, and elements of V are denoted by letters v,w,..., with or without subscripts, then a unit h is a set of actions $h(m_0) = v_0$, $h(m_1) = v_1$ ..., $h(m_r) = v_r$, interpreted as simultaneous performance of these actions. The class of all such units will be denoted by H.

One comment here is about the status of "non-action" # , i.e. of omission, or pause. It will be interpreted accordingly to the medium to which it refers; thus, # on medium of utterances means silence, on medium of gestures means lack of gesture, on medium of facial expressions means lack of expression, etc.

Another comment needed here is that it ought to be clear that the set H is very large, and comprises units which are admissible, together with units which are not admissible. Thus, it becomes necessary to eliminate from H the units which are not admissible and define the subsets of actions which are appropriate for particular media. This will be done by using the fourth primitive concept, namely that of language of communication actions L.

The formal construction is exactly the same as in mathematical linguistics. One considers namely the class of all finite strings of elements of H, to be denoted by H*, and then identifies the language of actions L with a subset of H*. The only difference is that in this case the "strings" are composed of units and the units are multidimensional. Therefore a "string" is really a matrix, or a "bundle" of parallel strings, one on each medium. This justifies the term "multidimensional" in reference to the language of communication actions L.

Thus, each string of units has the form

$$u = h_1 \ldots h_n = \begin{array}{cccc} v_{01} & v_{02} & \cdots & v_{0n} \\ v_{11} & v_{12} & \cdots & v_{1n} \\ . & . & \cdots & . \\ v_{r1} & v_{r2} & \cdots & v_{rn} \end{array}$$

where v_{ij} is the action on medium m_i in j-th unit, i.e. $v_{ij} = h_j(m_i)$.

Multidimensional language of communication actions

Formally, L being a subset of H*, is a class of matrices of the above form; elements of L will be interpreted as admissible strings of units.

Naturally, since the set L of admissible strings of units is a primitive concept, one cannot say explicitly which strings are in L and which are not. That is, one cannot describe a formal procedure which would distinguish admissible strings from non-admissible ones.

It should be quite obvious why such a procedure is impossible to give : it is impossible, despite many attempts, even for a much simpler structure, namely for the natural language.

Thus, the programme here is more or less similar to that in analytical models of linguistics : to take the concept of language, i.e. set of admissible strings, as a primitive one, and rely on common intuition in distinguishing admissible strings from non-admissible ones (perhaps agreeing that L may be a fuzzy set). Then instead of trying to find a grammar which would generate this set, try to develop useful tools for the analysis of this set.

In the case under consideration, each "string" being in fact a matrix of actions, describes a fragment of communication behaviour, taking into account all what is happening on all media during the considered time interval.

The intuition of language of communication L, or equivalently, of admissibility of a string of units of communication actions, is such that a given string (matrix) is admissible, if it is physically possible to perform, and also if it represents a fragment of behaviour which is meaningful, socially acceptable, etc.

It is not necessary to give more exact interpretation, mainly because the flexibility of this interpretation gives the model more applicability. For instance, if one wants to describe and analyse some communication actions of special type, such as royal court protocol, Japanese tea ceremony, etc., the set L (or : admissibility) has to be appropriately defined in each context (conforming to court protocol, etc).

Admissibility of a unit, and language of a medium

The preliminary formal definitions, expressed in terms of L are the following :

$H' = \{ h \in H : \exists u = h_1 \ldots h_n \in L \text{ such that } h = h_j \text{ for some } j \}$
$L_i = \{ v_1 \ldots v_n : \exists u = h_1 \ldots h_n \in L \text{ such that } h_j(m_i) = v_j \text{ for all } j \}$
$V_i = \{ v \in V : \exists u = h_1 \ldots h_n \in L \text{ such that } h_j(m_i) = v \text{ for some } j \}.$

Here H' is the set of all admissible units, i.e. set of all possible i-th rows which appear in "matrices" from L; next, L_i is the set of all possible i-th rows of matrices from L, i.e. set of all strings on medium m_i which may occur. Finally, V_i is the class of actions which may appear on medium m_i.

The two immediate consequences of the above definitions are :

Proposition 7. If $u = h_1 \ldots h_n \in L$, then $h_i \in H$ $(i = 1, \ldots, n)$.

Proposition 8. If $h \in H$ and $h(m_i) = v$, then $v \in V_i$ $(i = 1, \ldots, r)$.

These theorems assert that an admissible string of units must consist of admissible units only, and that an admissible unit must have, on each medium, an action which is admissible for that medium. These are, of course, very tautological consequences; it is of some interest, however, to remark that the converses to Propositions 7 and 8 need not be true. To illustrate why it is so, consider the second proposition (the situation with the first being analogous). One can imagine a unit which is composed of actions such that each of them is admissible on its medium in some context, but it may be impossible to perform them simultaneously. For instance, an utterance on verbal medium is an admissible action for this medium, and so is sticking out a tongue on the medium of facial expressions. But these two are incompatible together.

The individual languages L_i, as well as the whole language L of communication actions may be analysed in terms of linguistic concepts, such as L_i-equivalence, or L_i-parasiticity (resp. L-equivalence and L-parasiticity), as given in definitions 5 and 6.

Both these concepts describe the constraints of syntactic character, and are rather standard in linguistic contexts. However, owing to the fact that one deals with "multidimensional" language, one can define concepts, which describe "intermedial" constraints between actions. The general idea is to analyse which actions on a given medium are implied, and which are excluded, by the fact that some actions are performed on other media.

Let us put $B_i(v) = \{h \in H : h(m_i) = v\}$; then, $B_i(v)$ is the set of all admissible units which have action v on medium m_i.

One can now define the relations of enforcing and exclusion as follows :

Definition. Let $u \in v_i$, $v \in V_j$. Action u enforces action v (u ! v) if $h \in B_i(u) \Rightarrow h(m_j) = v$; action u excludes action v (u § v) if $h \in B_i(u) \Rightarrow h(m_j) \neq v$.

Thus, in the first case, any unit with an action u on medium m_i must also have action v on medium m_j, while in the second case it cannot have action v on medium m_j.

It ought to be mentioned that these relations are of syntactic character : both enforcing and exclusion means that if any unit h has some property, then it must (cannot) have some other property. Breaking any of these rules would result in a unit which is outside the class of admissible units.

From these definitions it follows that we have :

Proposition 9. Relation of enforcing in transitive, and relation of exclusion is symmetric.

In general, the knowledge of the relations of exclusion and enforcing provides information about the structure of the communication units in a given situation which one wants to analyse.

10.2. *Semantics*

Let us now turn to the last two primitive concepts of the system, namely the set S of meanings, and the function f describing (fuzzy) meanings of strings of communication actions. Formally, the function f maps the set $L \times S$ into the interval $[0,1]$, i.e. it assigns to every pair (u,s) consisting of a string of actions and a meaning, a number f(u,s), which represents the degree to which string u means s.

Since the language of communication actions L contains, among others, all utterances, the set S must contain all possible meanings of utterances.In this paper, however, the main object of interest will be those meanings which are carried mostly by non-verbal media. Examples of such meanings are "friendliness", "scorn", "dislike", "annoyance", etc.

Most of these meanings have the property that they may appear in modalities; thus, the set S will contain meanings such as "mild annoyance", "annoyance", "extreme annoyance" etc., forming a linear scale.

The meaning of a string u need not be unique : there may be more than one element s such that $f(u,s) > 0$.

Meanings of units, and empirical interpretation of f

The assumption of existence of the function f imposes certain restrictions on the possible interpretation of elements of S and L.

It ought to be clear, that such a broad concept as a function assigning meanings to whole strings (in a fuzzy or nonfuzzy way) would be of a limited value, if it were not accompanied by the assumption that meanings are also assigned to particular units in the string. Indeed, only having the meanings of units separately, and also the meanings of strings of these units, one can express various possible ways in which a particular action on a given medium contributes to the meaning of a unit, and in which a given unit contributes to the meaning of the whole string.

Thus, it will be assumed that function f assigns meanings also to admissible units.

The assmption of existence of fuzzy membership function f requires the clarification of two points :

— to which extent, and how, it narrows down the possibilities of choice of units ?

— how to interpret the concept of fuzziness in the context of the present considerations ?

Regarding the first question, the answer is that in order to get the proper interpretation, one cannot distinguish units which are too small : each unit must be sufficiently large so as to carry some meaning.

Regarding the second question, namely the interpretation of fuzzy membership function f, there are several alternatives in interpreting the fact that $f(h,s) = p$, that is "the degree to which the meaning of h is s equals p".

1) Firstly, it may mean that p is the fraction of persons, who — when confronted with unit h — claim that its meaning is s.

2) Secondly, s may be one of the possible meanings of h, and p is the frequency of occasions in which it is used to denote s.

3) Thirdly, p may be the degree of certainty (of a given person) that the unit h was used, in a particular occasion, in meaning s.

4) Finally, there is also possible fourth interpretation, connected with the following conceptualization of unit h.

Generally, h consists of some utterances, that is, action on the verbal medium, and a number of actions on other media. Now, these media concern gestures, facial expressions, and so on, and there is a certain degree of arbitrariness of performing a gesture, within the limits imposed by the topology of the human body in the surrounding space. Formally, it could perhaps be described by an appropriate parametrization of h, that is, by treating h as a family of units, say h_z, where z runs through a parameter set. Different h_z differ in the degree to which some gestures, expressions, and so on, are performed. As z changes, the units h_z change somewhat their meanings. A typical example is when a gesture looses its meaning when it is exaggerated, that is, when parameter z assumes the extreme value.

In this interpretation, the function f would represent the fraction of values of z for which the meaning s is associated with h_z.

Regardless of the interpretation of the values of the function f, one can use it to express the role played by various actions on particular media in contributing to the meaning of a given unit, as well as the role played by a given unit in contributing to the meaning of a string which contains this unit.

The first of these problems will be discussed in some detail in the next section. The discussion of the second problem will be omitted; the formal definitions are analogous in both cases.

The role of an action for meaning of a unit

In general, the definitions will be based on comparison between meanings of two units, differing only on one medium.

Let $h \in H$ be a fixed unit, and let h_i^v, h_i^w,... denote the unit h modified so that actions v, w, .. are performed on medium m_i, and the remaining actions are unchanged.

One can now say that in context h, the action v expresses the meaning s stronger than action w, if

$$f(h_i^v, s) > f(h_i^w, s),$$

The above definition can be applied to get more subtle classification, if one used the concept of the "neutral" action" # .

We may namely introduce the following definitions :

In context h, action v

— *supports* s, if

$$f(h_i^v, s) > f(h_i^{\#}, s),$$

and in particular, it *generates* s, if

$$f(h_i^v, s) > f(h_i^{\#}, s) = 0;$$

— *inhibits* s, if

$$f(h_i^v, s) < f(h_i^{\#}, s),$$

and in particular, it *cancels* s, if

$$0 = f(h_i^v, s) < f(h_i^{\#}, s);$$

— is *neutral* for s, if

$$f(h_i^v, s) = f(h_i^{\#}, s),$$

and in particular, is *irrelevant* for s, if

$$f(h_i^v,s) = 0 = f(h_i^{\#},s).$$

The supporting of meaning occurs if adding action v increases the degree of expression of meaning s above the level which would have been if the pause were in place of v. In particular, in the latter case, if the meaning s would not be present at all one may say that v generates s.

The other definitions are based on the same principle : inhibition occurs, if replacing pause by v decreases the degree to which s is expressed, and cancellation — if this decrease is complete (to zero).

A typical example of supporting, for the meaning such as "friendliness" might be the accompanying smile. In a similar way one can easily produce examples for other concepts

The definitions above covered the case of one action on one medium. The situation becomes more complex when one considers the joint effect of two or more actions on different media in supporting, generating, etc, of a meaning. The details may be found in Nowakowska(1978), and will be omitted here.

10.3. *Pragmatic semantics*

At the end, one more conceptual construction will be given, provisionally called "pragmatic semantics". Intuitively, the problem here may be described as follows. Imagine that one is restricted in his expressions only to some medium or media, such as in ballet, or for deaf and mute persons, where medium of utterances is not allowed; in silent films, where one has only medium such as facial expressions to his disposal (at least in some scenes); in radio programmes, where one has only verbal medium, without facial expression, and so on.

The question is then of possibilities of expressing some meanings, or differentiating between some other meanings, with the use of only the permitted media of expression.

Thus, one can distinguish a fixed subset of the language of communication L, say $L' \subset L$, such as "ballet language", etc. For a given meaning s, one can then define two subsets of L':

$$L^+(s) = \{ u \in L' : f(u,s) > 0 \}$$
$$L^0(s) = \{ u \in L' : f(u,s) = 0 \}.$$

Thus, $L^+(s)$ is the set of all strings in L' which express the meaning s in some positive degree, while $L^0(s)$ is the set of all strings in L' which do not express meaning s at all.

Using these sets one can define the following concepts from "pragmatic semantics". Of course, all these concepts are relative to the selected language L', i.e. the chosen set of media for expression.

First of all, given two meanings s and t, one may say that s is embedded in t, if $L^+(s) \subset L^+(t)$. Next, s and t are *inseparable*, if $L^+(s) = L^+(t)$, and *incompatible*, if $L^+(s) \subset L^0(t)$ (hence also $L^+(t) \subset L^0(s)$). Finally, s and t are *orthogonal*, if $L^+(s) \cap L^+(t) \neq \emptyset \neq L^+(s) \cap L^0(t)$.

The first concept, of embedding, corresponds to the usual concept of inclusion of meanings in semantics. Here, however, the inclusion is due not to the content of s and t, but rather to the limitations imposed by the necessity of using only certain media, but not the others.

As regards the second concept, inseparability, it corresponds in semantics to the concept of partial synonymy : when the meanings overlap, expressing one of them means automatically expression of the other also, at least partially.

The third concept, of incompatibility, is perhaps closest to the concept of negation — or, more precisely, entailment of negation of one meaning by the other.

Finally, the last concept, of orthogonality, corresponds to logical independence.

These concepts utilize only part of information contained in function f(u,s), namely the fact that the value is zero or is positive. This is why they may be termed "presence-absence concepts". It is also possible to express some concepts which utilize full information contained in these function. They may therefore be called "degree of expression" concepts.

For that purpose, denote

$$L_a(s) = \{ u \in L' : f(u,s) \geqslant a \}$$

i.e. the class of all strings of units which express the meaning s in the degree at least a.

One can now define *perfect synonymity* by the requirement that $L_a(s) = L_a(t)$ for all a (thus, s and t are perfectly synonymous, if $f(u,s) = f(u,t)$ for all u).

Another concept is that of a-supporting : t is a-*supported* by s if

$$\exists a_0 \ \forall a \geqslant a_0 : L_a(s) \subset L^+(t).$$

This concept is related specifically to fuzzy meanings, and has no direct counterpart in usual semantics. It is especially connected with two facts :

1) that meanings can be expressed in varying degrees (which, as should be stressed, is not the same as saying, that varying degrees of a meaning can be expressed);

2) that an expression — unit of communication actions — may express several meanings at once.

The concept of a-supporting describes the situation when an expression of a given meaning, if it is enough clear or unambiguous, (i.e. if the meaning is expressed in sufficiently high degree) must involve the expression of some other meaning too.

Supporting of meaning is quite an interesting phenomenon, as it shows the interrelations between meanings imposed by the restrictions due to certain media. The meanings as such may be conceptually distinct, but because of the limitations of the media, one cannot be expressed without the other. This set of relations between meanings shows therefore the extent of "blurring" due to various media.

To use some examples, consider the difficulties encountered by actors in silent movies, who had to convey meanings by facial expressions only, and had to differentiate between, say surprise and fear, or hate, jealousy and anger, etc.

Finally, the last problem which it is necessary to mention here is that of interchangability of media, that is, expession of the same meaning on one medium instead of the other.

Here the basic definitions are, briefly, as follows. First of all, one has to consider strings of units involving actions on one medium only (or on a given set of media), i.e. consider situations when some media are "banned". Generally, let u(i) stand for a string which involves only one medium m_i, i.e. such string in which all media except m_i are "banned".

One may then say that the meaning s is *expressible* on m_i, if

$$\exists u(i) : f(u(i), s) = 1,$$

i.e. if one can express s in full degree using only medium m_i.

Let $Q_i(s)$ be the shortest length of the string u(i) which expresses s in full, if s is expressible on medium m_i.

Given two media, the ratio $Q_i(s)/Q_j(s)$ measures the relative "dilution" or "contraction"' for media m_i and m_j, for meaning s.

To use an example, a dialogue in which the man says "Let us go for a walk", and the woman replies "Oh, leave me alone" may take several seconds if verbal medium is allowed. The same meanings expressed in a ballet scene may take some minutes; and in medium of facial expressions only, it is not expressible at all, at least the man's part, since no facial expression can convey with any reasonable lack of ambiguity a suggestion of a walk.

10.4. *Some empirical hypotheses*

It ought to be clear that the conceptual scheme outlined in the preceding sections is sufficiently rich to permit defining also other concepts. The obvious question one can raise is about the possible role which such a system might play for the analysis of the basic problem, namely that of relations between verbal and non-verbal media. It seems that this role could be summarized in form of three points :

1) the conceptual scheme allows unification of the empirical findings accumulated until now within a uniform classificatory scheme, providing the means for comparison of various systems of communication;

2) Secondly, it leads to directing and coordinating the future research, by aiming it precisely at study of those aspects which would provide the most relevant data for fitting the future findings within the present scheme;

3) Thirdly, it allows to formulate a set of empirically testable hypotheses.Some of these hypotheses, or simply, research questions, may be the following :

Hypothesis 1. The relation of enforcing occurs in general more seldom than the relation of exclusion.

The intuitive justification of this hypothesis lies in the fact that a given action on one medium necessitates one of several possible actions on other media (and not only one action). In such cases, a number of actions becomes excluded, but none is enforced.

Hypothesis 2. The ratio of number of cases of enforcing to the number of cases of exclusion depends on the pair of media. This ratio is highest (i.e. there is relatively high number of enforcings) for pairs connected with body movements. These enforcings are induced, at least partially, by constraints imposed by the human body.

Hypothesis 3. There are more actions which generate meanings than those which cancel a meaning. In general, there is probably few actions with the latter property.

Hypothesis 4. Head, body and hand movements are probably more frequently used for supporting the meanings expressed verbally, than for generating a meaning.

On the other hand, facial expressions are probably more often used for generation of meanings.

Hypothesis 5. Voice pitch,intonationand speech disruption are the media on which it is easiest to express the meaning contradicting the utterance (e.g. through exaggeration, etc).

4. *Discussion*

The suggested theorie is, of course, far from being closed, and may be developed in numerous directions, such as a construction of a general semiotic calculus, based on some laws of perception of signs, or comprising interactional aspects of communication, hence the construction of multidimensional theory of dialogues. Finally, the theory may also be applied to natural language, by restricting the attention to verbal media only (utterances, intonation, etc). One could also think of a multidimensional analysis of written language, where the variety of type (italics, boldface, etc) play the role of various media.

At the end, it is worth while to sketch briefly an alternative approach to semantics of multidimensional language of communication actions.

From the point of view of the receiver, the situation is such that he observes sequentially a sequence of units, and assigns to it a meaning, often in a preliminary way, before the string is completed. This may be compared with sequential testing of hypotheses, where elements of S play the role of hypotheses, and the value of f(u,s) play the role of the degree to which a hypothesis was confirmed.

Successive steps in "testing" the hypothesis about a given meaning are based on the principle of approximate reasoning, in which particular units play the role of premises, and the meaning is a conclusion.

The formal foundations of such reasoning may be built as follows.

First of all, for the formal analysis, the premises and the conclusions have to be expressed as propositions, i.e. to represent verbally the units of communications and their meanings. The basic primitive concept here would be that of admissibility of a conjunction of propositions.

Having the concept of admissibility, one can define the semantic implication as follows : A (a set of sentences which describe a string of communication units) implies the meaning s, if the conjunction "A and not s" is inadmissible.

This, of course, is a standard way of defining implications; if admissibility is replaced by truth, one gets material implication; with impossibility, one gets strict implication; finally, by replacing inadmissibility by inacceptability of epistemic of doxastic grounds, one gets epistemic or doxastic implication of Hintikka (1962).

If one assumes that admissibility is a graded concept, such a construction wuuld lead to definition of function f for a given string u and meaning s : the value f(u,s) would be related to the degree of admissibility of the sentence "u and not s".

This allows to connect the approach suggested here with the theory of fuzzy sets (see Zadeh 1965). This theory has in particular interesting results concerning the numerical representation of such linguistic variables as adverbs "Very", "Seldom", etc.

On the other hand, the approach suggested here, in view of the fundamental assumption of discriminability of semantic variables on particular media, enables us to combine semiotics with a methodologically mature domain, such as psychophysical scaling. Basing semantics and semiotics on this discipline might give an access to new empirical foundations of these theories

Finally, the last important application of the suggested system lies in the possibility of forming new dydactics of teaching foreign languages (by combining teaching with means of expression characteristic for a given culture), and in improvements of methods of teachings deaf and mute, etc. Moreover, it can lead to a convenient classification and way of thinking about any interactional situation, such as clinical, guidance, managerial, etc., useful in developing new methods in social training.

It is also possible to obtain new theoretical applications of the presented system of communication in the domain of semiotics, through an introduction of kind of semiotic calculus (see Nowakowska 1978).

Finally, it is also possible to extend the formal theory of dialogues (see Nowakowska 1977), enriching it by the consideration of "multidimensional" languages of each participant. In the theory of dialogues cited above, the main problem was to formulate the conditions under which a pair of parallel strings of actions (consisting of utterances and periods of silences, when one listens to the partner) forms a dialogue — i.e. belongs to the "multidimensional languages of dialogues.

It was possible to give several necessary conditions of this sort, some of them of syntactic, and some of semantic character.

In the class of syntactic conditions, two main categories were distinguished : retrospective and prospective ones, where the constraints lie solely in the preceding (resp. following) part of the dialogue.

The semantic conditions were expressed in terms of fuzzy semantics; they assert, roughly speaking, that a subsequent utterance must belong, in a degree exceeding a certain limit, to an appropriate fuzzy set defined through the preceding utterances

These concepts were successfully applied to the theory of eristic dialogues, where the latter were represented as positional games, in which each of the utterances ("moves") causes a change in the "position" of the opponent in the semantic space.

It was also possible to outline a new theory of semantics, treated as a hierarchy of languages built over the alphabet of semantic markers, with the main mechanisms being those of generating, inhibiting, supporting, cancelling, etc of a given meaning (see definitions in section 10).

In consequence, the considerations of this section give a new approach to cognitive representation problems, leading to a form of semantic calculus, and connecting the problem of semantics and semiotics, with the possibility of using also the theories of psychophysical scaling.

11. *Final remarks*

What was shown in this paper is only a fragment of a very rich theory, and one of its numerous applications.

The idea of looking for syntax and semantics of the behaviour, for unification of verbal and non-verbal behaviour in one system, seems to be not only natural and intuitive, but also receiving increasing amount of attention, mostly due to its methodological consequences, especially in the social sciences.

Perhaps it is worth to point out that the idea of extension of the concept of language to nonverbal actions, put forward in the book "Language of Motivation and Language of Actions", appeared several years earlier than the similar program announced by Piaget on the Congress of Psychology in Paris in 1976.

The presented system allows not only for the analysis of human behaviour in terms of constraints imposed both by reality and by the cognizing mind, but also extends the way of thinking about a man as a decision maker, to the analysis of structure of pre-decisional situations, together with the study of utterances in situations of planning, explaining and justifying the decisions. These utterances constitute a linguistic representation of motivation, analysed logically and semantically in Nowakowska (1973), leading to principles of approximate reasoning, based on semantic admissibility.

The general theory is not closed; it is developed and applied to new domains and phenomena (e.g. prosocial behaviour; see

Nowakowska 1977a). The newest development (see Nowakowska 1978a) consists of a novel way of construction of semantics of language of actions, or, more precisely, on "reversing the order" of construction : it appeared possible, and in some respects fruitful, to start from building a theory of events, understood as sets of "histories" of a system, and then superimpose the actional structure on it. Also, admissibility of an action — hence also the language of actions, as a set of strings — is treated as a fuzzy concept. Consequently, one can distinguish a family of languages of actions, each corresponding to a specific level of admissibility, and superimpose this structure on the structure of goals, which may also be attainable to some degree. Such a construction provides an unusually rich structure, in which it is possible to express various rather subtle concepts pertaining the attainability, means of attaining goals, etc.

REFERENCES

K.J. ARROW, 1963, Social Choice and Individual Values. Wiley, New York.

N. CHOMSKY, 1963, Formal properties of grammars. In : R.D. Luce, R.R. Bush and E. Galanter (eds) Handbook of Mathematical Psychology, Wiley, New York.

J. HINTIKKA, 1962, Knowledge and Belief. Ithaca, Cornell University Press.

M. NOWAKOWSKA, 1973, Language of Motivation and Language of Actions, Mouton, The Hague.

M. NOWAKOWSKA, 1977, Towards a formal theory of dialogues, Semiotics, 17, 4, 291-313.

M. NOWAKOWSKA, 1977a, A model of prosocial behaviour, Polish Psychological Bulletin, vol. 8, nr. 3.

M. NOWAKOWSKA, 1978, Verbal and nonverbal communication as a multidimensional language. In : T. Borbé (ed.) Proc. of Second Semiotic Symposium, Vienna 1976. (in press).

M. NOWAKOWSKA, 1978a, Application of systemic approach in the social sciences, In : R. Trappl (ed.) Handbook of Cybernetics (in press).

L.A. ZADEH, 1965, Fuzzy sets. Information and Control, Vol. 8.