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THE ELEMENTARY THEORY OF COLLECTIVE ACTION

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1. *Concrete Examples*

Let us give some examples of what we call in this paper "collective action" in order to be clear about the topic we are analysing.

CA1 : Two or more persons carry together a heavy burden : all of them are in *contact at different places with the same material object (a stone, a piece of furniture) and all of them exert on this object physical force by means of their body movements, in such a way that the object is displaced in a given direction, realising by doing this a* purpose shared by all of them (they wish to remove this stone from the road or they wish to move the piece of furniture from one appartment to another).

CA2 : The staff of a bank, during working hours, present in the bank building, performs hundreds of different co-ordinated *actions* (telephoning, writing out contracts, receiving clients, paying out sums, taking inventory asf), alone or under continuous or discontinuous supervision, so that the overall purpose of the bank operation (profit for the proprietors of the bank, by offering various needed services to the many members of the public using it) is realised. Here there is only exceptionally contact with the same physical objects; the employees often do not see or know each other; the overall purpose is different from the individual purposes of the different employees.

CA3 : In September 1939, England declares to be in a state of war with Germany. Parliament has convened and has voted a bill; the Cabinet has come to a decision and certain diplomatic services have been directed to communicate these decisions to the responsible representatives of the "Third Reich". Here the concept of *representation* is basic; a state representing a nation, and served by state officials communicates "its" (collective) intentions to another state, representing another nation, represented by analogous officials.

CA4 : In the middle sixties, in certain cities of the USA, negro citizens, engaged by the brutality of white policemen, run through parts of the town, burning the cars of whites and destroying their shops (things they would not have thought of doing alone or on purpose). Here there is *no common aim, nor any formal representation or group decision but simply spontaneous mob action by contamination.*

CA5 : A group of people belonging to the philosophy department of a given university meet regularly to discuss the theory of observation, trying to work out an alternative to logical empiricism. The group is not a mob; it has no formal existence; no physical action is exerted; no organisation exists but *common problem solving with a partly* definite, *partly vague aim takes place.*

CA6 : An orchestra under the guidance of its conductor plays the fourth Symphony of Gustav Mahler. The *collaboration* of the composer whose work lays before the members of the orchestra, the performers whose repetitions made them familiar with the music and the conductor realises the Symphony for the enraptured audience.

CA7 : Two people take a walk together. A buyer and a seller agree upon the terms of a contract. Two persons get married to each other.

CA8 : A company of trained soldiers led by its captain, marches in front of the commander in chief, who gives them a formal salute. The movements of the men, of the captain and of the general are *rigorously coordinated in 3 different ways, prepared* by the earlier training of these soldiers of different rank.

These 8 examples are only a few among thousands. We have selected them however in order to point out that nothing mystical is meant by the term "collective action". We are all familiar with the phenomena described. We have moreover selected them in order to show how many different types of collective action we have to distinguish. The assertions we wish to defend are a) a small number of basic concepts from action theory are necessary and sufficient to describe all these forms of collective action, b) and all those forms of collective action, in order to be able to begin, to subsist and to reach success must apply a certain number of implicit or explicit rules that are the conditions of viability of collective action. It is this corpus of rules we are looking for. In this paper we shall only be able to present a defense of the first assertion. Later work will defend the second.

Our second basic assertion, concerning the rules implicit in the very concept of collective action does not exclude the existence of a multiplicity of types of collective action. The conceptual framework we are going to explain should show its usefulness by being able to express the specific differences between our eight examples of collective action.

An action is done by an *actor*, producing a *result*, with a given *purpose*, using certain *instruments* and *materials* The various theories of action that have been formulated stress each of them some of the relations between these five action elements, leaving others in the background. In earlier work we tried to overcome this unilateral stress of action logicians. We can not say however that, at the moment of this writing, our suggestions have been taken up. For this reason we shall have to be satisfied with developing a theory of collective action, starting from a theory of individual action that is itself restricted. We shall consider a) the collective action theory developed from an action theory stressing primarily the relation between the actor and the result of his action, b) the collective action theory in the framework of a theory stressing primarily the relations between the actor and his aims.

2. *The formal framework*

Stig Kanger, Helle Kanger and their pupils Lars Lindahl and Ingmar Pörn are in the first place responsible for this work, undertaken under the impulse of Stig Kanger (Ref. 1,2,3,4,5,6).

Let us consider a language L, containing the following signs :

1. Propositional variables and constants : p, q, r and p_i, q_i, r_i
2. Variables for actors : x, y, z and constant names for actors a_1, a_2 a_n
3. The universal and the existential quantifier : A and E; and the identity =
4. The logical constants of propositional logic and of class logic (negation and complement, conjunction and intersection, implication and inclusion, ordered set and the relation "being an element of").
5. Variables and constants for functions : f, g, f_1, g_1.

This language will have as axioms a) a set of axioms for propositional logic, b) a set of axioms for class logic, c) a set of axioms for first degree functional logic, d) a set of axioms for identity.

We shall introduce action by a very poor axiom system for the operator "Do". "Do (a,p)" is to be interpreted as : "the agent a realises the truth of the proposition p".

Only two axioms are added to the ones already mentioned in order to give some basic properties for "Do" :

(1) $(Ax)[Do(x.p) \rightarrow p]$: for all agents it is the case that if an agent does p, then p is true.
(2) (a rule) "If it is proved that p entails q, then it is proved that Do(x,p) entails Do(x,q)".

The "Do" operator is analogous to the "d" used by Henryk von Wright in "Norm and Action" as an operator applied to transformations "pTq" (the state in which p is true is transformed into the state in which q is true). The formulation presented here is essentially the one presented by Lars Lindahl (p. 66-68) (with modifications of terminology and a weaker action rule 2).

It is obvious that as an axiom system for action logic this far weaker version of Von Wrights T calculus is inadequate. We are not interested here in the problems of action theory as such however. We have to go forward to the theory of collective action types.

In order to do so we need the concept of action type. The sentence (3) "(a,p)eT" means : the actor a and the state of affairs p stand to each other in a relation R belonging to the type T. For instance : a has done p, or a has not done p, or a can do p or a can not do p. Both in Lindahl and in Pörn's approach, deontic modalities are introduced and the relation type T may be the relation of being allowed to do p or being obliged to do p. We are however of the opinion that obligations and permissions are meaningless before we have at our disposal the concept of collective action. This is a strong difference between the approach we would advocate and the one followed by the Kanger school : we only introduce obligations after studying collective action. They do so before. As a second remark we notice that instead of limiting ourselves to introducing propositional calculus we want to introduce relation calculus.

The sentence (4) "Do(a,R(ab))" is allowed and we can say that an actor puts himself in given relations with other actors. In order to express in the same language the concept of action type we have to allow heterogeneous relations existing between actors and propositions (or between propositions and propositions).If we do so and if we use special terms for this type of relations we can express the following states of affairs (5) "Do(a,R'(ap))" and (6) "Do(a,R''(p,q))" : actor a puts himself in a given relation with the proposition p, and actor a realises a given relation between the propositions p and q. We can express ourselves in this way but we shall often continue to use the formulation proposed by Lindahl in his very complete treatise.

(7) "(a,p)eT" has one of the following forms : 1. Do(a,p). 2. Do(a,—p), 3.—Do(a,p). 4.—Do(a,—p). If, instead of introducing the operator "May" or "Shall" (as Lindahl does who develops the concepts of deontic logic before developing the concepts of collective action) we introduce another actor b, and if we consider the "Do" sentences as sentences in the same sense as others than we ob tain the following possible combinations (8): 1. Do(b,Do(s,p)), 2. Do(b,Do(a,—p), 3. Do(b,—Do(a,p).—Do(a,—p)), 4. Do(b,—Do(a,p)), 5. Do(b,—Do(a,—p)), 6. Do(b,(—(—Do(a,p).—(Do(a,—p))), 7. —Do(b,—Do(a,p)).—Do(b,—Do(a,—p)).

As "Do(a,p) → p", (9) Do(a, Do(b,p)) → Do(b, p).

The actor b does not preserve any autonomy whatever. In order to be able to reach a more realistic framework, Ingmar Pörn has introduced the S (or 'support') operator S(a, p) means : a makes p more probable (10) S(a, p) → p is false, but (11) (p → q) → [S(a,p) → S(a, q)] remains true.

It is interesting to interpret these possible basic interactions between two agents, but none of them constitute what we call collective action because in none of these cases mutuality, or reciprocity are present. For some collectivistic agent types (12), the "(a,b,p)eT" can not be analysed as a logical function expressible by boolean operators (13) of "(a,p)eT" and (14) "(b,p)eT". The fact that a *triadic* relation exists between a, b and p that cannot be so analysed as a logical function of any number of *dyadic* relations is one of the two steps one can take towards the development of a logic of collective action.

The other direction we are going to prefer however is the introduction of a "collectivisation operator". Lindahl (p. 214) tells us "by a collective agent is understood here any unit that may be chosen as an agent, formed by some operation on a set or an ordered m-tuple of people". Lindahl is aware of the fact that there exists more than one collectivisation operator (he indeed mentions three of them, but he does not attempt to obtain a systematic insight in the set of possible natural collectivisation operators). He is also aware of the fact that he is here breaking new ground, and that he should, while doing so, offer some natural and plausible characteristics for the collectivisation operator he prefers (whose properties, important as they may be, are (so we shall show) insufficiently analysed). We consider it out primary task, in this paper, to obtain a deeper insight in the complete set of collectivisation operators on the one hand, and in their *individual properties* on the other hand, in order to attempt to express the basic differences between the types of collective actions preserved.

3. *The Collective Actor.*

Let us then add to the language L, a new sign "+", *an operator who applied to the names of actors, yields again the name of an actor (a collective actor this time)* (15). "Do(s+b,p) "reads as follows "a and b do p together".

To the two axioms Ax 1 and 2 we add the following ones :

(16) 3. (Ax)[(x+y) = (y+x)]: when x and y do something together then y and x do it together : the relation 'doing something together' is commutative.

(17) 4. (Ax)(Ay)(Az) [(x+(y+z)) = (x+y)+z)]: when x does something with y and z together, then x and y together do something with z together. The operation "doing something together" is associative.

The other properties of identity and of Do remain formally the same but in fact their meaning is enriched because the variables for actors now range equally over individual and collective actors.

Some other candidates for the role of axioms about "+" are mentioned but Lindahl does not use them :

(18) (Ax)((x+x) = x)
(19) (Ax)(Ay)(Az)[(x+y = x+z) → (y = z)]
(20) (Ax)(Ay)(Az)[(x = y) → ((x+z) = (y+z))].

The proposal one might derive from the introduction of this collectivisation operator is that *the logic of collective action is the logic of the collectivisation operator* "+", taken in conjunction with the other specific operators of action logic. If this were the case it would be most important to be clear about the properties of "+".

We have to make immediately the following remarks

1. When two agents act together they are often not playing the same role in the action they undertake (the one might be leading, the other helping for instance). In that case, it is inadequate to say that "+" is commutative. We propose thus to consider both a commutative and a non commutative "together".

2. More important however is the following remark : when a married couple goes for a walk together with a friend, then this does not always imply that the husband and the friend go for a walk, together with the wife. Groups acting as groups can do things together with outside individuals without being broken up in the act of doing so (and on the other hand, the opposite may occur). For this reason we

must consider both an associative and a non associative "together". This remark is most important because the possibility of concerted action of groups as groups (without losing their structure) depends upon this feature not introduced by Lindahl.

3. It is thus obvious that we already have to consider *four* collectivisation operators : the first is *associative* and *commutative;* the second is *neither*, the third is *non commutative and associative*, the fourth is *non associative and commutative*.

4. The Formulae 18, 19 and 20 introduce interesting problems.

4a. Even if the actions performed by the group (x,y) are exactly those performed by the group (x,z) this does not entail that the actions performed by y alone are also identical to the ones performed by z alone (a counter-example would be the case where actors in group have systematically other abilities or preferences than actors alone). We could strengthen the condition and consider not only the identity of (x+y) with (x+z), but even state that with *all* possible partners, y and z behave identically. Still then it might be that isolated they act otherwise (and so, are not identical).

One could certainly answer that the identity clause does not only refer to actions but to all possible properties or characteristics. In that case we have no problems about (19), but then the properties considered lose all interest and do not belong to action theory as such. Analogous problems arise when we study (20). These questions are not academic ones; they concern the degree to which the individual actors are changed by entering into action-groups. It will be rewarding to develop collectivisation operators having and not having these characteristics. We surmise that when (19) or (20) are false, then (16) and (17) should also be false. This would rule out certain possible collective action logics but reveal important relations

5. We do consider that "x+x" is meaningless in any possible interpretation of a "togetherness" operator. The property "x+x = x" should thus not be either true or false, but absurd.

By modifying Lindahl's axioms, and multiplying his "+", the logic of collective action becomes concrete. Studying these purely formal questions we have already met the problems of subgroups (associativity), of hierarchy (commutativity), of the relations between individual and collective action. (The truth or falsity of $(x = y) \rightarrow [(x+z) = (y+z)]$).

4. *Individual and Collective Actors*

The most interesting properties of collective action will however be related to its connexion with individual action.

Let us compare

(21) Do (a+b, p) and[Do (a,q) . Do (b,r).]

It is certainly true that

(22) "Do (a+b, p) → Do (a,p) . Do (b,p)" is false.

Are however

(23) Do (a+b,p) → —[DO (a,p) . Do (b,p)] or even
(24) Do (a+b,p) → [—Do (a,p) . —DO (b,p)] valid ?

They are not so in general but only for specific cases. "a and b enjoy themselves together" or "a and b offer advise to c" are collective actions that are not incompatible with their individual counterparts.

As in the cases of (16) and (17) we need a collectivisation operator with the *strong* collectivistic property (what the actors do together, they do not do separately) and a collectivisation operator with the *weak* collectivistic property (sometimes, but not always, the actors do individually what they do together).

As L. Lindahl points out (p. 222, op. cit.) the need for collective action arises out of the fact that some collective actors are able to do things their individual counterparts are unable to perform. This can only be expressed with the help of *modal* operators.

(25) Can Do (a+b,p) . — Can Do (a,p) . — Can Do (b,p).

The following property is however certainly true :

(26) Do (a+b,p) → [(Eq)(Er) Do (a,q). Do (p,r)]

In general the inverse implication will be false and, moreover — [(q.r) → p]; this negative statement stands in relation to the strong collectivistic property but takes account of the fact that all collective action reflects itself in individual action (even if we cannot reduce it to individual action).

The study of the relations between individual action and collective action leads however to more specific results.

In order to state them, a reference to *time* is important. We are limiting ourselves to simultaneous relations.

(27) Do t_1 ((a+b),p) (Do t_1 means : at the moment t_1 the action occurs) → (Eq)(Er) Can Do t_1 (a,q) . Can Do t_1 (b,r) . [—[Do t_1 ((a+b),p)] → —[Can Do t_1 (a,q) ∨ Can Do t_1 (b,r)]].

The performance of the collective action gives the individuals participating in it, new action possibilities at that very moment. Two remarks have to be made : a. these new action possibilities, not necessarily realised, are thus not the individual actions implied by (though not constituting) any collective action, b. similar individual possibilities are opened up for later moments by the collective action. We do not, as announced, take up this topic however, at this point.

(28) Do t_1 ((a+b),p) → (Es)(Et) — Can Do t_1 (a,s) . —Can Do t_1 (b,t) . [— [Do t_1 ((a+b),b)] → [Can Do t_1 (a,s). Can Do t_1 (b,t)]].

The fact that agents engage in a collective action *deprives* them also at that moment of action possibilities they might have had if they had refrained from joining. In order not to trivialise both these assertions, we must add to this formula the provision that q ≠ s or t and r ≠ s or t (in sentences 27 and 28). The action possibilities the partners are deprived of, temporarily or definitively after performing the collective action are even more important than the action possibilities they add to their "repertoire" in consequence of their joint venture. We cann not rest with simply stating these strange properties, we must derive them from more fundamental attributes. At the present moment, this is an important open problem.

Participating in a collective action, the actors a and b must consider more combinations of actions than when acting alone. This increases the cost of planning with the chances of success, and, by providing help makes possible to risk more. They add supple adaptability, 2) But on the other side, "Am stärksten steht, wer allein steht", they lose self-reliance and must take the other into account (compromises).

5. *Representation and Delegation*

Continuing the study of the relation between collective action and individual action, we have to consider the following situations.
1. An individual a represents a group G if for all p, belonging to a

class P, and for all q belonging to a class Q, whenever a does a P-action, all members m of G, commit a Q-action (whenever a prime minister sends by wire a certain signal to another prime minister, the members of the armed forces of their respective countries engage in war-like activities).

(29) Repr (a,G) = (Def) (Ap)(Aq)[[(p ∈ P).(q ∈ Q)] → [Do (a,p) → (Am)[(m∈G) → Do(m,q)]]]

This concept of representative collective action has been considered by us at some time as the basic concept of collective action, in the sense that

(30) Do (a+b,p) would be definable by (Ex) Repr (x, (a,b)) . Do (x,p).

(where (a,b) is simply a set, and where x maybe either a, or b or any c). The method should certainly be pursued, but we deem it now more likely that the representation relation will only exist as a consequence of (G + a) acting together in the earlier sense.

6. *Large Collective Actors*

A generalisation of the problem occurs if we introduce the sign ±, a collectivisation operator that can be applied not only to two persons, but to n persons (with n finite but arbitrarily large).

The statement "Do (± (x_i), p)" reads as : "The x'_i's together do p). If we have associativity "$\pm(x_i)$" will be easily definable by "+" but without associativity, it will be more expedient to introduce the new operator as an independent sign, with axioms similar to those proposed for "+".

The following cases are important :

(31) (a) Do $(\pm x_i,p) \rightarrow [(Ej)\ (j < i)\ .\ Do\ (\pm x_j,p)\ .\ \hat{x}_j \subset \hat{x}_i\ )]$.

(32) (b) Do $(\pm x_i,p) \rightarrow [(Ej)\ (j > i)\ .\ Do\ (\pm x_j,p)\ .\ \hat{x}_i \subset \hat{x}_j\ ]$

(33) (c) Do $(\pm x_i,p) \rightarrow \neg[(Ex_i)\ (i = 1,2...n)(-\ Do\ (x_i,p) \rightarrow -\ Do\ (\pm x_i,p))]$.

(a) is true if whenever an n-group does something, a subgroup of it does it also. (b) is true if whenever an n-group does something, the group obtained by adding individuals to it, does the same. (c) is true

if there does not exist an individual belonging to the group, that can prevent a group action. (a), (b), and (c) taken together certainly describe the action of a mob, while the negation of (a), (b) and (c), taken together describes the action of an orchestra.
When more than two actors are introduced, specifically new problems arise.

For instance,

(34) "Do (a+b+c,p) . Do (a+b, p_1) . Do (a+c, p_2) . — [(Eq) Do (b+c, q)]"

is a possible situation : a can by talking on the phone with b, and simultaneously in his room with c, come to an agreement about a collective decision of the 3, without any interaction between b+c. In this case clearly (35) and (36) are true.

(35) Do (a+b+c, p) ≠ Do (a+(b+c), p).

(the formula at the left of ≠ is true, the one at the right is false).

(36) Do (a+b+c, p) → Do ((a+b)+(a+c),p)

But other cases can be considered in which neither formula (35) nor formula (36) are true.

Between the problem due to the presence of 2 actors, and those due to the presence of n actors, specific layers of problems exist for 3, 4, 5... actors. The exploration of this unchartered domain has yet to begin.

When thinking about John Von Neumann's theory of games and Jacob Marshak's theory of teams, where the members of a team all have the same purposes and the members of a game different ones, the theory of collective agents, that, untill here, neither presupposes identity, nor conflict of aims is a third possibility (weaker, but more general).

7. *Collective Action and Interaction*

Before introducing the collectivisation operator, we had already the means to express social *interaction*, using (37).

(37) Do (a, Do (b, p)).

This social interaction could also create social structure

(38) Do (a, Do (b, R (abc..., p)))

(where p could be missing). When we combine these methods of expression with the collectivisation operator we can consider the following statements

(39) Do (a, Do (b+c, p)) , meaning that

an individual makes the members of a group do something together.

(40) Do (a+b, Do (c, p)), meaning that

two actors together make an individual do something

(41) Do (a+b, Do (c+d, p)), meaning that

a group makes a group do something.
Note : important special cases are those where the initiating and the executing actors have elements in common.

(42) Do (a, Do (a+b, p))
(43) Do (a+b, Do (a, p))
(44) Do (a+b, Do (a+c, p))
(45) Do (a, R(a+b, c, p))

Sentence (45) means that an individual puts a group in relation to another individual, with reference to a proposition p.

(46) Do (a+b, R(c, d, p))

reads analogously as : a group puts two individuals in a certain relation to each other with reference to p.

Considering these statement types (39) — (46), we meet some plausible candidates for theorems.

(47) Do (a+b, p) → Do [a+b, R(ab)]
(48) Do (a+b, Do (c+d, p)) → Do (a+b, R (a,b,c,d)).

or even

(49) Do(a+b, Do (c+d, p)) → ((c = a v b) ∨ (d = a v b)).).

If a and b do something together, they create a relation between each other (47). If a and b make c and d do something together, they create a relation between a, b, c and d (48) (or even : the 2 groups must have common members) (49).

These two possible theorems can only be evaluated as correct or incorrect if a semantic model for "+" is introduced and a completeness theorem proved. We make a brief remark on semantic models later on.

The combination of the "Do" operator with the "togetherness" operator makes it plausible to add to the strong action "Do" the weak action, introduced by Ingmar Pörn : "S (a, p)" means (as mentioned before "a does something that increases the probability of p".)
"S(a, p) → p" is false, but
(49) (Eq) (S(a,p) → q) . [Pr (p/q) > Pr (p/—q)

is true (there exists a q, consequence of S(a,p), such that the probability of p given q, is larger than the probability of —p, given non q ceteris paribus). The function of the introduction of S is twofold.

(50) Do (a+ b, p) → (S(a, p) . S(b, p))

if a and b do something together, then each of them, taken separately, makes the common result more probable.
Moreover

(51) [Do (a+ b, p) → (Do (a,q) . Do (b, r))] → [S (a, Do (b, r)) . S (b, Do (a, q))]

If a and b do something together then a supports the action b does in realising q, and b supports the action a does in realising q. It is interesting to note that

(52) S (a, p) . S (b, p)

does not in general imply S(a + b, p) (given the fact that Pr (p/q) > r, and Pr (p/s) > r does not have as a consequence that Pr (p/q,s) > r. While it is easily understood that Do (a, p) . Do (b, p) does not imply Do (a+b, p) either, we can only (in the absence of a clear interpretation of Do !) derive this negative conclusion in the following way : let Do (a, p) reduce to a) Do (a, p) → q) and b) q → p. Let Do (a, q) reduce to c) Do (a, q) → r and d) r → p. Then the negative conclusion would only follow if from (q → p) . (r → p), (q.r) → p is not derivable. For material and strict implication it is derivable, but for entailment it is not (see Anderson—Belnap, Entailment). The question as to under what circumstances Do (a, p) . Do (b, p) entails Do (a+b, p) arises and should be answered.

(53) [Do (a, p) . S(b, Do (a, p))] . [Do (b, p) . S (a, Do (b, p)]

is a necessary, but not yet sufficient condition.

We have seen earlier that the discussion of two algebraic properties of "+" "(commutativity and associativity) has revealed important features of collective action. This leads us to expect that the discussion of distributivity, of iterations and of inverses will have an equally important impact.

8. *The set of collectivising operators and distributivity*

In order to be able to speak about distributivity however, we need to have more than one collectivisation operator. L. Lindahl (p. 224) mentions at least 3 collectivisation operators : 1. a does p after consultation of b, 2. a does p with the help of b. 3. a and b are joint parties (Kanger and Kanger 1966, p. 103) in producing p. The properties of these collectivisation operators have not been studied, and they are not worth being studied, as long as we do not have systematic principles of classification at our disposal : some such principles of classification are suggested by the very cases Lindahl mentions. 1)"a does p with the help of b" can be analysed as follows

(54) Do (a+b, p) . Do (a, Do (b, p)) . — Do (b, Do (a, p)) . S (b, Do (a, p))

both do p, but a makes b do it, while b does not make a do it, and b supports a's action.

2)"a does p after consultation with b" introduces both time and deliberation. At moment t_1

(55) Do (a, p) ∨ Do (a, —p)

is the case. At t_2

(56) S (b, Do (a, p))

is the case. Finally at t_3

(57) Do (a, p)

is the case. Neither of these three conditions is tautological : at t_1, — (Do (a, p) $\vee$ Do (a, —p)) might have occurred. We shall then express the type of collective action concerned as follows (introducing a temporal logic, similar to the one expressed in Resher—Urquhart's "Temporal Logic"Ñ

$$(58)\ \underbrace{[\mathrm{Do}(t_1)(a,p \vee \mathrm{Do}(t_1)(a,-p)]}_{1} . \underbrace{[S(t_2)(b, \mathrm{Do}(t_i)(i > 2)(a,p))]}_{2}$$
$$\underbrace{.[[1 . 2) \rightarrow_c \mathrm{Do}(t_3)(a,p)].[\mathrm{Do}(t_1 - t_3)(a+b, p)]]}_{3}$$

Note : It is only possible to give these analyses if we do not demand that "+" be commutative. If we want to impose this restriction, we have to refrain, in the two last definitions, from asking that Do (a+b, p).

We do not claim that this analysis could not be improved upon, by introducing non extensional operators (like "with the purpose of" or "aiming at" or "believing"). A few remarks on this topic will be made later on. But, as stated before, we wanted to see how far we could go in the analysis of collective action without leaving the field of extensional operators.

We did analyse in our own way the two examples Lindahl gives (without studying them further) because we wanted to show the type of considerations that might lead to a natural classification of collectivisation operators.

1) When two actors do an action together they may a) act in the same place at the same time, b) in different places at the same time, c) in the same place at different times, d) at different times and at different places.

2) when two actors do an action together they may a) make each other do it, b) or either a or b might make the other do it (without a symmetrical influence on the other side) or neither of them might make the other do it.

3) what has been said about making each other do something might also be said about supporting each other in doing the collective action.

4) when two actors do something together they may do the same or different things.

5) when two actors do something together they may interact in the beginning of the action, or at the end or in the middle or

continuously. We say that two actors interact if event caused by one of them cause modifications in events caused by the other. Interaction may be symmetrical or unsymmetrical.

We still do not have any reason to claim that this is a complete list of binary coöperation types. But is now already possible to see that the complex cases mentioned a few instants ago have to be build up out of the simpler categories mentioned here. Moreover it is also already possible to ask some distributivity questions.

Let Do $(a+_1 b, p)$ mean that a and b make each other cooperate. Let Do $(a+_2 b, p)$ mean that one of the two influences the other, while Do $(a+_3 b, p)$ means that none of them influences the other. This allows us to state the following conjectures.

(59) Do $((a+_1 b) +_2 c, p) \rightarrow$ Do $((a+_2 c) +_1 (b+_2 c), b)$

(60) Do $((a+_2 b)+_1 c, p) \rightarrow$ Do $((a+_1 c) \vee (b+_1 c), p)$.

(61) Do $((a+_1 b)+_3 c, p) \rightarrow$ Do $((a+_3 c) +_1 (b+_3 c), p)$.

The difficulty of the distributivity theorems is that we did offer some suggestions towards the natural classification of binary collectives while distributivity introduces ternary collectives.

We leave the problem open at this point however in order to complete our analysis in three directions : a) we ask in what sense we could express duality theorems, b) we ask what relations could be discerned between Do (a+b, p) and the logical connnectives that may occur in p (conjunctions, disjunctions, implications, negations) ? c) We ask for what reason acting together occurs and what are the non extensional operators involved in expressing these reasons.

9. *Inversion theorems*

We can translate "a does p without b" as

(62) Do (a — b, p) meaning
(63) Can Do (a+b, p) .–Do(a+b, p) . Do (a, p).

The sentence

(64) Do ((a+b) — b, p)

means "a and b together, without b, do p, and thus is either absurd, or entails that "a+b" is responsible for p.

Here it is the place to introduce a third meaning of collective action (the first and main type is expressed by the various collectivesation operators, the second by the *representation relation).* One can say that a and b do p together, if some agent c reacts towards a and b as if they were coactors of p. Formally this would be expressed as follows

(65) (1) (Ax) [(Ez) Do (z, p)   Do (x, P (z))] .
     (2)(Ey) [Do (y, P(a)) . Do (y, P(b))].

P(z), P(a), P(b) mean that z, a, b possess the property P. The definition states that all actors confer upon the actor doing p, the property P (and only upon this actor). Moreover there is an actor who confers both on a and on b the property P.

The difficulty of introducing a collective agent is here avoided *by making the other agents perform by their behavior the collectivisation operation.*

We think that interesting relations exist between collective action as *representative action*, as *collective performing* and as *common responsability* but here we want only to draw attention to these features in a side remark. Our main purpose is still the search for duality theorems.

Let us define, for any given actor, two anti-actors.

(66) 1. "Ant_1 (a) = b" = (Def) (Ap) S(a, p) → S(b, —p).

(we cannot use "Do" in this definition, because Do (a, p) → p would lead us to the contradiction "p. —p)").

(67) 2. "Ant_2 (a) = b" = (Def) (Ap) Do (a, p) → S (b, —(Ex) Do (x, p)).

The first antagonist seeks to counteract anything a does, the second antagonist seeks to destroy any actor doing what a does. These definitions allow us to form

(68) Ant_1 (a+b) and
(69) Ant_2 (a+b).

Ant_2 (a+b) seeks to destroy anything doing what (a+b) does, and this can mean either destroying the partners or only the partnership (when

(70) c = Ant_2 (a+b) . — (c = Ant_2 (a)) . —(c = Ant_2 (b))).

(71) Ant_1 (Ant_1) (a)

supports anything a does while a universal supporter of a need not be an

(72) Ant_1 (Ant_1).

To the contrary a universal supporter of a well be an

(73) Ant_2 (Ant_2)

of a but not inversely an

(74) Ant_2 (Ant_2)

a universal supporter. Looking for De Morgan theorems, let us compare

(75) (a+b) (Ant_1(a) + Ant_1(b), Ant_1 (a+b), and Ant_1 (Ant_1(a) + Ant_1(b)).

Let P be the set of actions of a+b. Let P_1 be the same for a, P_2 for b. Ant_2 (a+b) counteracts all these actions. Ant 1a + Ant_1b is a group, counteracting together all the individual actions of a and b ($P \neq P_1 \cup P_2$, $(P \cap P_1)$ may be empty or not, $(P \cap P_2)$ may be empty or not). Ant_1(a+b) is an actor (individual or collective) counteracting the collective actions of a+b. Certainly

(76) $(Ant_1(a) + Ant_1(b) = Ant_2\ (a+b)$

is false. But under certain conditions

(77) $[c = (Ant_1(a) + Ant_1(b))] \rightarrow (Eq)\ S(c,q)\ .\ Do\ (Ant_1\ (a+b),q)$
(78) $[c = Ant_1\ (a+b)] \rightarrow (Eq)\ (S(c,q)\ .\ Do\ (Ant_1(a) + Ant_1(b)\ q)$

What are these conditions ? It is not our purpose here to develop this calculus more completely, but we conjecture that the truth of 1 and 2 entail a stronger type of collective action than their falsity.

10. *Relations between the complexity of the actor and the complexity of the action.*

When we compare

(79) Do $(a+b, p_1 . p_2)$
(80) Do $(a+b, p_1 \vee p_2)$
(81) Do $(a+b, p_1 \rightarrow p_2)$ to Do (a+b, p)
(82) Do $(a+b, p_1 \Leftrightarrow p_2)$
(83) Do $(a+b, -p)$

we can distinguish two types of collective actors :

A. *Diffractive* collective actors are such that if they do a complex action, the different individual actors present are more specially connected with a part of the complex action. In the limit this leads to strange distributive laws of the following forms

(84) Do $(a+b, p_1 . p_2) \rightarrow$ [Do (a, p_1) . Do (b, p_2)]
(85) Do $(a+b, p_1 \vee p_2) \rightarrow$ [Do$(a,p_1) \vee$ Do(b,p_2)]
(86) Do $(a+b, p_1 \rightarrow p_2) \rightarrow$ [Do $(a, p_1) \rightarrow$ Do (b, p_2)]
(87) Do $(a+b, p_1 \Leftrightarrow p_2) \rightarrow$ [Do $(a, p_1) \Leftrightarrow$ Do (b, p_2)].

Weakly diffractive actors have laws of the form

(88) Do $(a+b, p_2 . p_2) \rightarrow S(a, p_1) . S(b, p_2)$.

Collective actors that are stable, will be *weakly diffractive.* (This property can only be expressed in a combination of temporal logic and action logic).

Collective actions are *essentially centralised if the negation of the* strong diffractive properties hold (for instance

(89) Do $(a+b, p_1\ p_2) \rightarrow -$Do $(a, p_1) -$Do (b, p_2)).

They are *simply centralised* is neither the strong diffractive properties nor their negations hold. We can not claim that all collective actors are diffractive, or that all of them are centralised. But future theoretical development will certainly have to take into account the consequences of the combination of diffractive with diffractive, centralised with centralised and diffractive with centralised collectivities.

11. *Specifity of our approach*

Now that we have explored, developing and modifiying Kanger's, Pörn's and Lindahl's ideas, the possibilities of an elementary calculus of collective action, it is not superfluous to note the correspondences and differences between our different approaches.

The Scandinavian philosophers are mainly interested in law, we are mainly interested in social action. This explains why they introduce essentially (Kanger and Lindahl more consistently than Pörn) deontic logic, while we have avoided it alltogether. We must credit Pörn with the idea of introducing Do (a, Do (b, p)) and the weak S-operator, to Kanger and Lindahl we owe the "+" operator. To ourselves we must attribute the idea of combining the Do, S, and + systematically (in this following leads by Lindahl), and, the more systematic exploration of different types of "+" operators, with the purpose of investigating distributivity, duality and the relations between the logical connectives before the comma (in Do (a, p)) and those after the comma. Moreover, we refused to have only commutative and associative collectivisers, and showed the sociological importance of this refusal. We indicated — without being able to give a full account — that those among us whose methodological individualism could not agree with the Do (a+b, p) calculus could reduce it to a calculus of representative action, or to a calculus of collective responsible action.

12. *Collective Action and Deontic Operators*

Even after introducing these modifications one remain far away from what we need in order to answer our main question : "*What are the rules any collective actor has to apply, in order to start, continue and succeed in his collective action ?*" We shall presently introduce some non extensional ideas entirely absent from the Kanger tradition, but we want to prepare later remarks by noting that deontic logic can be introduced in our present framework, not independently, but as a derived theory. Different methods may be used.

(90) Op = (Def) (Ax) (Az) Do (x,p) $\rightarrow$ Do (z, Do (x,p)).

(so also : Do (x,p) $\rightarrow$ Do (x, Do (x,p)) - by substitution). A weaker version might be :

(91) (Ax)(Az) Do (x,p) $\rightarrow$ S (z, Do (x,p)).

A more operational, but more punitive version is :

(92) [— (Ex) Do (x,p)] $\rightarrow$ (Ez) (or (Az)) S (z, Do (x,p)).

p is permitted (P(p)) can be defined as —p is not obligatory.

2. A second type of definition (we personally prefer this second type to the first) is :

(92) O (Do (x,p)) = a) (Az)(Aq) Do (z,q) → Do (z,p)

(where z ranges only over the universe of dual, ternary or n-ary collective actions). (the obligatory actions are those implied by all collective actions).

(93) b) (Az)(Aq) [S(z,q) → q] → Do (z,p)

The succesfull actions imply the obligatory actions :

(94) c) (Az)(Ex)(Ey) (z = x+y) . ((Eq)(S (z,q) → q) → (Do (z,p)) :

the obligatory actions are those that are performed by all *successfull* (i) *collective* (ii) *actors* (iii).

We ask the reader to keep these possibilities in mind and now we proceed to introducing briefly some non extensional concepts.

13. *The Intensional Theory of Collective Action*

In order to define completely collective action we need *epistemic logic* :

(95) If Do (a+b, p)

is true, a must believe that b has the realisation of p as a purpose, and b must believe that a has the realisation of p as a purpose. In a stronger version, both a and b must know this, and must know that the partner knows it.

Any collective action is done by a large or small, durable or short lived organization. We can thus readily accept that the theory of collective action must join the theory of organisation. The theory of organization has already a long history. Recently however it has become clear that organizations are task executing open systems whose contact with the environment both depends upon the technologies used to execute the different tasks and upon the models the various agents have of this environment. These models, in as far as they are relevant to the operation of the organisation, can be represented by graphs mapping the causal models of the environment. This has been done very concretely in "Cognition in Organisations" (by M. Bougon, K. Weich and D. Binkhorst). This paper is extremely important from the point of view of those who wish to synthesise epistemology and collective action theory for different reasons : a) the cognitive state of n agents is represented by a graph (representing causal relations), b) the collective agent itself can be represented by a graph, mapping the social network), c) the following 4 questions can then easily be asked : 1. How does the locus in the social graph of a sub-agent factually determine his causal graph? 2. What is the optimal causal graph for a sub-agent at a given locus? 3. How can the n cognitive sub-graphs be aggregated into one collective cognitive graph ? 4. How can the collective action purpose and means be derived from the form of the cognitive graph (individual or aggregated).

We propose in future work to correlate, in a graph, the actors to points, and the togetherness relation to arrows. A collective agent will then be a set of interconnected points that, in a condensation of the first graph, has a neighborhood (in going and outgoing arrows) identical to or sufficiently similar to, the neighborhood of the points it is the condensation of (a condensation of a graph is a new graph where the points correspond to sets of points of the earlier one).

The collaboration of action logic, belief logic and graph theory is a necessary and promising step.

2. In order to define collective action we need the *concept of purpose* (as was already apparent in 1). Here we meet an important problem. While it is clear that the causal impact of two actors can by interference exert a common force (this is the justification of Do (a+b,p)), it is also clear that no collectivity has mind, self awareness or purpose. If

(96) Ma (p, q)

means (we take Chisholm's symbol) "actor a realises p in the course of his effort to realise q", then M (a+b)(p, q) is, taken literally, absurd. M (a+b) cannot exist as (a+b) has no selfawareness. Still, we might demand that Ma (r, p) and Mb (s, p) be true, and add that when these conditions are satisfied, and when certain additional conditions we are going to formulate are met, then the formula

(97) M (a+b) [[Do (a+b,p)], q]

is true (we stress the necessary difference between this formula and the one we rejected, because it entailed the hypostisation of an abstract).

The interaction between Do [(a+b), p] and M (a+b) [Do ((a+b), q] is of great importance (we abbreviate M(a+b) [Do (a+b), q] by M [Do (a+b, p), q]. We can for instance consider

(98) Do t_1 ((a+b, p) → Mt_n [Do (a+b,p), p]
(99) Mt_1 [Do (a+b,p), q] → —[Mt_n Do (a+b,p), q]

When at t_1 a partnership does p together it will at t_n do this with the purpose of doing it.

When at t_1, a partnership does p together with purpose q, it will at t_n not do p together with the purpose q. Both assertions can be joined if

(100) Mt_n [Do (a+b,p), p] → —Mt_n [Do (a+b,p),q]

(then 2 is a consequence of 1). A third assertion is also interesting :

(101) Mt_1 [Do (a+b,p),q] → Mt_n [Do (a+b,q)r]

Here the purpose becomes a means, as before a means became a purpose. The concept of "alienation" can only be expressed by means of the interaction between the extensional and the intensional version of collective action. The assertions (98 — 101) are however much too crude. Alienation should be introduced gradually, and in a context allowing us to overcome it, and indicating how to do so.

Other interesting concepts can also be introduced if epistemic logic is combined with the logic of collective action.

We did already make use of time logic in earlier pages. By combining epistemic logic with time logic we can conceive the concept of "role". A role is a set of actions, co-actors believe that their partners will perform, when they engage themselves in a collective action (or more generally a set of properties co-actors believe their partners to have when they participate in an undertaking). Formally stated :

(102) Rôle(b) = (Def) [$\hat{P}$] Do (a+b,p) → B(a, P(b))]

the set of properties P such that when a and b do p together, a believes b to posess these properties P.
Considered with reference to time, we can distinguish the future, present or past role as follows :

(103) Fut. Role (b, t_n) = (Def) [$\hat{P}$ Do (t_o — t_r) (a+b, p) → Bt_n (a, Pt_{n+j} (b))]

the set of properties such that when a and b do from time t_o to t_r, p, a believes at t_n that b will posess between t_n and the end of the action at t_r (n+j must have all possible values between n and r). These properties P

3. In order to define collective action we need (as we saw) modam logic. In order to show this once more we are going to describe a very strong type of interaction.

(104) $Do_{int}(a+b,p)$ entails that

a) if a did not do p_1, b would not be able (would not have the possibility) to do p_2.
b) if b did not do p_2, a would not have the possibility to do p_1.
c) p_1 taken together with p_2, causes p.
d) it is possible that Do (a+c, p) or Do (c+b, p), but only if in the first case Do (a, p_3) and Do (c, p_4) (where it is not necessary that $p_1 \Leftrightarrow p_3$ or $p_2 \Leftrightarrow p_4$), and if in the second case Do (c, p_5) and Do (b, p_6) (under the same conditions).
e) a and b know all this, and this *knowledge* is one of the *causes* of what they do.

These very stringent conditions express the fact that the two co-actors determine each other's possibilities and, knowing this, adapt their own efforts to the strategies of the partner in order to reach the common aim (the fact that this adaptation occurs is described by the possibility to realise p by means of different coalitions whose members have however to change their own strategies in function of those of their new partners).

For reasons of simplicity we take the modalities used here to be physical or causal modalities (neglecting the problem of practical modalities we have studied in earlier work).

It is important to realise that *subjunctive conditionals* are used to state that actors acquire different abilities when changing from one coalition to another (and a fortiori, when abandoning the zero-coalition — acting alone — for a non zero one).

Condition (104) can be weakened in many ways : a can be true without b, b can be true without a, d_1 can be true without d_2 or d_2 without d_1 (with only some of the non equivalence clauses satisfied). But in one of these weakened versions it must be true, for collective action to occur.

Note : Another type of weakening is the following one : "Replace a by "If a did not do p_1, b would not do p_2" (even if the possibility was preserved).

We stress that the complex "+(int)" to be defined by the existence of mutual beliefs, mutual purpose and mutual adaptation of actualities and virtualities is a new notion. (completely absent from the Scandinavian approach).

(105) Do (a+(int) b, p) $\neq$ Do (a+b, p)

We have said *nothing* about its properties, and can not do more here than introduce it. We recommend the study of its many versions as a necessary step in the development of the theory of collective action. The problem of passing from two person teams to n person teams will be more complex for "+(int)" than for "+".
Note : Without going into these matters, we should be allowed, once we have either Do (a+b , p) or Do (a+ (int) b, p), to form the expressions

(106) B (a+b, p)
(107) K (a+b, p)
(108) P (a+b, p/—p)
(109) O Do (a+b, p)

(the collective actor a+b believes p, knows p, prefers p to —p, is obliged to do p). If we are allowed to form these expressions, we should inquire of the properties of collective belief, knowledge, preference, obligation are basically different from those of their individual counterparts ?

14. *Concrete Examples and Formal Theory*

We are now going to conclude this paper by two remarks(R_1 and R_2).
R1 : the concepts introduced are sufficient to distinguish the eight examples of collective action briefly described in the beginning.
CA1 : Do (a+b, p) $\rightarrow$ Do (a, P (c)) . Do (b, Q(c)) : the two agents confer properties to the same object simultaneity and difference in location can be added by indices. If body contact is involved Do (a, P_n (b)) . Do (b, Pr (a)) can be used.
CA2 : Do (a+b, p) $\rightarrow$ [(Do (a, p_1) $\rightarrow$ Do (b, p_2)) . (Do (a, q_1) $\rightarrow$ Do (a, q_2)] : These conditional dependencies, expressing coordination, are true for many p_i and q_i and one of these p_i and q_i is always realised. Moreover M[[Do(a+b),p]q] $\rightarrow$ [— M[Do (a, p_x), q] . — M [Do (b, p_y), q] : the collective aim is not pursued by any individual agent. (or, in weakened versions : by *all* individual agents).
CA3 : The concept of "representation" analysed in this article, and expressed by formula (29) is specific for this case.

CA4 : As stated already, the fact that all members of the mob do ± the same thing, that n more or less members does not change their behaviour that there is *no coordination (see CA2), that there is no* collective or individual purpose, and that isolated they would not do the same describes this case.
CA5 : The vagueness of the common aim (yet existent), the specific role played by each actor, the absence of other actors having prepared the group and the inexistence of representation are specific for this collective agent whose aim contains, among other aspects, the modification of the beliefs of the members and the *agreement* of the members on the same beliefs.
CA6 and CA8 are characterised by the fact that individual and collective actors have *made* this orchestra and this army, and that the differentiated actions of its members have other individual and collective actions as aims.

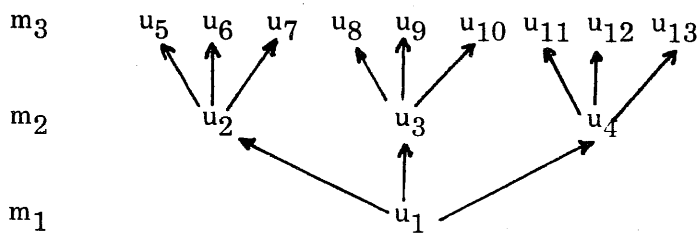
If M(p, q) . M(q,r) . M(r,s) .. is an action chain; if M(p,q) . M(r,q) . M(s,q) or M(q,r) . M(q,s) . M(q,t) are action rays (divergent or convergent); if divergent and convergent rays can be combined; if we have chains of rays, and rays of chains, then if for all terms of these conjunctions (or for some, or for many) the actors are systematically different, we reach the formula for the orchestra or the army company.
The reader can fill in the description of CA7 himself.
We claim moreover that our concepts, rich enough to characterize our examples, are also not *too* rich. They are *needed* to reach their specificity. We should, with the help of empirical sociology, multiply and systematise our classification. Part of this work, whose length exceeds the normal limits of a paper, is done.

15. *The Semantics of Collective Action*

R2. The major problem that remains is the development of a semantic model. We follow here hints of Von Wright.

Let there be n individuals $\langle I_1 \ldots I_n \rangle$ Each individual can at every moment $\langle m_1 \ldots m_r \rangle$ perform an action $\langle a_{11} \ldots a_{1m}, a_{21} \ldots a_{2l} \ldots a_{n1} \ldots a_{nn} \rangle$. This action leads the universe from one of its states to another $\langle u_1 \ldots u_k \rangle$ is the set of states.

The action tree of an isolated individual has the following form (let there be 3 moments, and let the actor have only 3 actions at his disposal). We suppose each time that the individual acts alone.

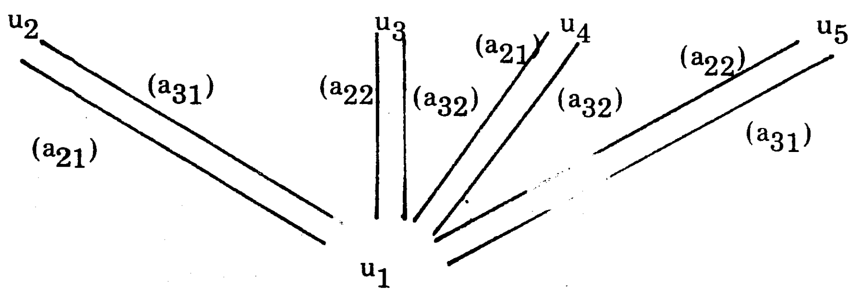


A history is a path from bottom to top, intersecting each level only once.

We have as many such trees as theire are individuals. A model of collective action will be a specific combination of several trees. First. Interaction is introduced by considering trees whose nodes are joined by more than one arrow.

Let there be 2 actors 2 and 3 and let each have 2 actions.

1) The *interaction* tree has the following form : Each pair of nodes is joined by the same number of arrows (not performing a specific action is performing another specific action). As the universe-states are not all different, it is possible for different arrow combinations to reach the same universe states



If no simultaneous action of different actors is allowed then, if we introduce a multiplicity of arrow *types*, we can let each actor act after the other.

2) Interaction does not yet imply collective action. Collective action is represented by a graph. The edges of which correspond to n-tuples of edges of an interaction graph, or to a sequence of edges of n graphs. The interaction of individual action and collective action is represented by a superposition of graphs, at least one of which is a collective action graph.

We take an interaction tree, select a combination of actions of 2, 3 or n actors as one action of a collective actor. We repeat this selection for as many collective actors as we wish to introduce and then draw the new tree combining individual actions with collective ones.

If collective actors are formed and eliminated at given moments, we shall have trees in which not all nodes are joined by the same number of arrows.

In this model, the building of a new actor is represented by an operation combining a specific bundle of actions of individual agents into a unique arrow that is an action of the collective agent. The only general restriction is that whenever an A_i (collective act) has been absent at a given level, after having been present earlier, it cannot occur later.

This paper has made many proposals that should lead to a diversity of formal systems for use in the analysis of collective actions; but the main task is the mapping of the adequate semantics on the adequate syntax We understand that at the present moment work on an analogous model is done by D. Batens (Univ. of Brussels) and co-workers).

16. *Conclusion*

Within the limits of this paper, we cannot show the usefulness of this approach. But we stated our aims. By developing a general theory of collective action we want to discover general conditions of viability, suboptimality and optimality. These conditions will have impact on politics, ethics, the sociology of science and of art, and show the possibility to analyse them from a unitary point of view. The unifying potential of the theory of collective action has already been shown by Maria Nowakowska (see her theory of collective action in this volume). We consider a study of the relations between Nowakowska's approach and our own as very rewarding.

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