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TRUTH DETERMINANTS. ANOTHER THEORY OF MEANING FOR THE PROPOSITIONAL CONNECTIVES

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The aim of this paper is to show that the truth-functional theory of meaning of the propositional connectives could be contrasted with another related, so-called truth-determinantal theory. With both theories and their ensuing different logics at hand we understand better the inadequacy of truth-functional logic and the difficulties in trying to make up for its shortcomings.

1. *Introduction*

When raising the question about the philosophical relevance of the concept of meaning, the answer is in a sense both immediate and impressive. For as Michael Dummett, in particular, has pointed out[1], there is, according to Frege, one part of philosophy that is the basis on which all other disciplines rest, viz. the theory of meaning. This has not always been the view of philosophers; Descartes, for instance, and many of his followers saw epistemology as the fundamental theory on which all other forms of philosophy should be based.

But if we are prepared to take the Fregean approach and to admit instead theory of meaning as the starting point of philosophy there are two things to note. First, this order of approach will never prevent us from moving on from theory of meaning to questions about what we know and how to justify it. Merely we should be clear about the meaning of epistemological words before entering into a territory that difficult.

Second, being clear about the meaning of a philosophical interesting word is very seldom, if ever, the same as to be able in language, ones own or in another language, to state the meaning of the word in question. Rather let us again follow Dummett in looking upon such a philosophical analysis of meaning as an attempt "to make explicit what had been only implicit in the learning process. The deeper it goes, the more it is concerned with levels of language at which verbal explanation plays a minimal part in the introduction of expressions; at which we acquired an understanding of expressions by learning in practice to employ them rather than by being told how to use them."[2] For such purpose of analysis we have to work with models of meaning, to find out how language actually does achieve its communicative function.

This view of philosophy brings it into close contact with a science that originally belonged to philosophy, but by now is considered as a self-contained discipline by many : logic. For logic is concerned with analysis of meaning of the so-called logical constants; Aristotle started this project by his analysis of "all", "some" etc. in the classical theory of the syllogism, and the Stoics supplemented his efforts by their famous propositional logic. Also a modern logician such as Tarski will be of the opinion that "to establish the meaning and usage [of such terms as "not", "and", "or" and "if-then"] is the task of the most elementary and fundamental part of logic",[3] propositional logic, that deals with the logical constants we call propositional connectives of which the four mentioned are the main ones. Surely to trace the consequences of this meaning in detail is the business of the logician. For the philosopher, on the other hand, what is at stake is to find a model for the meaning of the connectives, if possible a general model to cover them all, that will make us understand how these terms function in language.

Without here[4] trying to defend this view, I propose that such a model, to be satisfactory, should not ignore the close relationship between meaning and truth; these concepts are certainly not identical as some philosophers have been inclined to think.[5] On the other hand, nor are they unrelated as much loose talk about 'meaning' and 'use' by other philosophers of language could make us believe. Meaning and truth are essentially related in this way : the meaning of a proposition is its truth conditions and the meaning of parts of the proposition such as words, e.g. the propositional connectives, are given by the way they contribute to the truth conditions of the proposition of which they are a part.

2. *Truth functions versus truth determinants.*

One model of meaning for the propositional connectives apparently conforms so well to the criteria of meaning analysis stated in the introduction and is so well-known and established that a longer explanation is hardly necessary.

According to this model the connectives of propositional logic for truth functions, and following Quine "a compound is called a truth function of its components if its truth value is determined in all cases by the truth values of the components".[6] That is to say that the truth conditions of a compound is fully specified in terms of the truth value of the constituents in such a way that the compound will be true under certain combinations of truth values of the components, otherwise false. For the connectives in question their truth conditions are shown in the following familiar truth-tables for a negation, a conjunction, an alternation and a conditional :

p	~ p
t	f
f	t

p	q	p . q	p v q	p ⊃ q
t	t	t	t	t
t	f	f	t	f
f	t	f	t	t
f	f	f	f	t

It follows from the way the truth-conditions are specified that such propositions as "2 × 2 = 5 or New York is a large city" and "if France is in Europe then the sea is salt" are true by the mere truth-value of the constituents; both propositions are taken from elementary textbooks of logic,[7] and there used as example of what we have to accept as true doing logic; similar propositions, of course, could be picked out from any textbook on propositional logic as a system of truth functions.

Nonetheless it cannot be denied that the concept of a truth function could be used as a basis for a theory of meaning for the propositional connectives that is both general and clear. Moreover, this theory furnishes us with a logic that is claimed to be sound and complete, as a matter of fact even a decidable logic. It seems as if we could want nothing better. However, it is generally admitted that besides the textbook propositions quoted above, there are other paradoxes or at least paradoxlike consequences of this logic. For the moment this must suffice as an excuse for contrasting this clear theory of meaning for the connectives with another related one, but hardly as clear; we can then later by help of the new theory try to show why truth-functional logic is bound to contain paradoxes and even, in a certain sense, to be unsound.

This concept, to distinguish it from a truth function, I propose to call a *truth determinant*. A compound is a truth determinant of its components when the truth of the compound is compatible with some truth values of its components and excludes others such that any possible combination of values is either allowed or ruled out and such that no single combination, in general, is sufficient for the truth of the compound. If we say, for instance, "to-morrow we will go to the movies or to the theatre", the truth of this alternation is compatible with going to the movies and not to the theatre, not to the movies but to the theatre, and perhaps (dependent upon the sense of 'or') to both places, whereas the possibility that we do neither is excluded.

The two concepts of a truth function and a truth determinant are alike in seeking the truth values of compound and components as dependent upon each other. They also both assume the principle of bivalence, i.e. that a proposition, whether compound or component, is either true or false, though it does not seem to be a consequence of this principle that the truth value is always knowable. The main difference between the two concepts is that it is the truth determinant that is basic and determines the values of the components, not the other way round as the truth-functional approach will have it. Thus the parts are dependent upon the whole rather than the whole a function of its parts. Clearly not only alternations such as the example above, but also conditionals are easily looked upon as truth determinants. So are conjunctions and negations, though they for reasons we shall see in the next section form a special category. On the other hand, there are a number of compound propositions e.g.. "I believe that the weather will be nice to-morrow" that are no more truth determinants than truth functions. Such examples confirm that besides the truth-functional realm there is another one, viz. the truth determinantal where also nothing but truth values and relations between these count, before we enter into the realm of meanings.

3. *Further clarification of the notion of a truth determinant.*

Unfortunately it must be admitted that the concept of a truth determinant is not as clear as we could wish it to be. In particular, it seems that the definition in the preceding section by speaking of allowed truth values of the compound stated the necessary condition for the truth of such a compound and also, by its last requirement, stated what was *not* a sufficient condition. Nothing, however, was said as to what positively would constitute the sufficient condition, and until that is supplied we have only a partial understanding of the concept of a truth determinant. On the other hand, the new concept seems to be sufficiently interesting and promising to be worth an attempt of clarifying this and other points.

First, let us ask if the concept of a truth determinant is possible at all ? This may seem to be a strange question the moment it has been introduced and contrasted with the concept of a truth function. However, as fine a logician as Lukasiewicz was of the opinion that if we accepted the principle of bivalence, there could be only one kind of conditional, the truth-functional one[8]. This is certainly also true if we read the truth table above in the ordinary truth-functional way. Nonetheless it seems to be possible to stick to the principle of bivalence, but at the same time let the table define a truth-determinantal conditional. In itself the table is nothing but a schema that has to be explained in order to be understood, and we can make this explanation in such a way that a conditional different from the truth-functional one will stand out. Perhaps this will be most conspicuously seen if we examine Frege's way of introducing the concept of conditionality.

In 1879 Frege gave in his *Begriffschrift*[9] not only an original treatment of multiple generality; besides thus founding quantification theory he also gave the first axiomatic version of propositional logic. However, in the explanation of the meaning of the propositional connectives there is an ambiguity right at the beginning, in Frege's account of the conditional. To be sure, this ambiguity does not last long. Frege ends it by opting for what is definitely a truth-functional sense of 'if-then'. Had he not done so a different theory of meaning for the connectives could have been the result, and logic might have taken a different turn.

Frege begins his fifth paragraph by saying, in effect, that if 'p' and 'q' are propositions, there are the following four possibilities :

(1) 'p' is affirmed and 'q' is affirmed
(2) 'p' is affirmed and 'q' is denied
(3) 'p' is denied and 'q' is affirmed
(4) 'p' is denied and 'q' is denied

He then introduces his sign for conditionality, where we use the more common horseshoe ' ⊃' and say that 'p ⊃ q' stands for or means that the second "of these possibilities does not take place, but one of the three others does." But this formulation is ambiguous and leaves us with an open question. What is the case if one of the three possibilities actually does take place ?

Either we can say with Frege that the conditional is made true by occurrence of such a combination of truth values of the components and thus interpret the conditional as a truth function. Or we can refrain from thus taking a particular combination as sufficient for the truth of the conditional. If we favour the latter course, but stick to Frege's idea about what the conditional tells us, we are bound to interpret the conditional rather as a truth determinant and to have at the same time the answer to our question about the sufficient truth conditions for a conditional in terms of truth values of the constituents.

Frege explicitly says that the conditional *means* that one of three possibilities does take place. That is to say that the three possibilities also, in some way or another, must be sufficient for the truth of the conditional. These possibilities, however, are not related in such a way that the occurrence of one will lead to truth of the conditional. Consequently they must be related in another and stronger way. There must be a connection between these possibilities such that the truth values in question together with the appropriate relation between them furnish the necessary and sufficient condition for the truth of the conditional.

We can express the difference between the truth-functional and truth-determinantal conditional by making the necessary distinctions in the following definition :
if p then q $=_{df}$ ‘p and q’ is true, or ‘not p and q’ is true, or ‘not p and not q’ is true.

If, in this definition, we take ‘or’ as a weak truth-functional ‘or’, we will define the truth-functional conditional. Let us suppose, however, that there is also a stronger, as a matter of fact a truth-determinantal, ‘or’ in language, then the stronger ‘or’ will connect the truth possibilities and thus define the truth-determinantal conditional.

In symbols that should be self-explanatory the two definitions will look like this :

$$p \supset q =_{df} (p \cdot q) \vee (\sim p \cdot q) \vee (\sim p \cdot \sim q)$$
$$p \rightarrow q =_{df} (p \cdot q) \veebar\!\!= (\sim p \cdot q) \veebar\!\!= (\sim p \cdot \sim q),$$

actually exactly like but for the ‘ $\veebar\!\!=$ ’ (here not to be identified with exclusive alternation) that emphasizes a stronger condition for the truth of the alternation than that at least one of its terms is true, a thing that would make the first alternation and thus the corresponding conditional true. That both kinds of ‘or’ occur in language is illustrated further below, if there should be any doubt about this.

In both definitions the meaning of the conditional is given by its truth conditions in terms of truth values of its components, presupposing the ordinary truth-functional tables given above for the sign of conjunction ‘.’ and negation ‘~’. A truth-functional conditional is true for some combinations of truth values of its components and false under another. The truth-determinantal conditional likewise is true for the very same combinations, provided there is a connection between these, and false otherwise.

From the difference in truth conditions between the two conditionals there follows also a difference in the conditions of falsity. '$p \supset q$' is false if and only if '$p . \sim q$'; '$p \rightarrow q$' is false if '$p . \sim q$' as well, but not only if so. For it may also be false if the connection between the truth cases fails. Consequently a false truth-determinantal conditional is compatible with any combination of truth values of the components. Conversely, however, though a particular combination of truth values leaves us with no knowledge of the truth of the truth-determinantal conditional, the combination that is denied, in Frege's words, '$p . \sim q$' will be enough for inferring the falsity of the conditional.

If we take a look at the usual truth table for the conditional as above, it seems that the truth-determinantal could be extracted from it, if only we allow ourselves a certain freedom in reading it. Thus reading from left to right we further stipulate that only a connection between the first, third and fourth possibility of truth values of 'p' and 'q' will secure a 't' in the column for the conditional. On the other hand, assignment of 't' to 'p' and 'f' to 'q' will bring out '$p \rightarrow q$' as false; so here, as in all sufficiency conditions of falsity, the truth-determinantal reading will coincide with the truth-functional. They do not, however, coincide with respect to necessity conditions of falsity; if '$p \rightarrow q$' is false, it will not necessarily lead to '$p . \sim q$' as stated above. But we know from the truth of our newly defined conditional that one of the three other possibilities will take place as Frege said. Defining the truth-determinantal conditional by the truth table thus brings in the concept of connection or dependence between the truth cases and further requires a certain asymmetry in reading it. Nonetheless the definition seems to be in conformity with the table.

Turning now to alternation — non-exclusive alternation — we have also here one definition of the truth-functional alternation, and a different one of the truth-determinantal alternation :

$$p \vee q =_{df} (p . q) \vee (\sim p . q) \vee (p . \sim q)$$
$$p \veebar q =_{df} (p . q) \veebar (\sim p . q) \veebar (p . \sim q)$$

Both definitions are unavoidably circular. The latter stresses that a truth-determinantal conditional will never be true merely by the truth of one of the combinations of truth values in the definiens, but will require for its truth a connection between these possible combinations.

If, however, we model a definition of a truth-determinantal conjunction upon the previous definitions, a curious thing will happen :

$$p \cdot q =_{df} p \cdot q$$

There is in this case no connection between truth values of the components to be taken into account by specifying the truth conditions of the conjunction. So if the particular combination where both components are true occurs, the conjunction will be true by this alone. Accordingly a conjunction may be said to be a truth determinant that logically behaves just like the corresponding truth function, or, in other words, with respect to conjunction the concept of a truth function and a truth determinant will coincide. This is not to say, however, that it would be impossible by other means to define a conjunction where the conjuncts are related to or connected with each other. But such a concept would be in conflict with the attempt of looking upon the truth-determinantal theory as a general theory of meaning for the propositional connectives. For from this general theory and its implications for the truth tables it follows that a conjunction will connect two propositions that need not be related or connected with each other, whereas a conditional and an alternation will always require connection.

Much the same as was said about conjunction could also be said about the last connective to be analysed. '$\sim$', the sign for negation. If we want it covered by our general theory it seems that negation must behave truth-functionally, though, perhaps, from other points of view it would be fruitful to distinguish between different concepts of negation.

The most important thing to note about the truth-determinantal definitions above is the occurrence of '$\veebar$' that is expressing a relation, dependence or connection between the components of the compound. This sign is an undefinable basic sign for a truth-determinantal approach. Of course, we could have used equally well the sign of conditionality '$\rightarrow$' since '$p \veebar q$' is equivalent to '$\sim p \rightarrow q$'. And since 'if-then' perhaps is the phrase we immediately associate with dependence and connection we should perhaps have done so. This dependence is a very general notion that like the notion of identity or partial order have special cases or interpretations within different fields of knowledge; to mention but a few, we may say "if the sun is shining, it will be hot" and here speak of a physical connection; "if the sun is shining, the sun is shining" emphasizes a simple logical relation, and "if the sun is shining, I will take a walk" informs that, by a decision, my behaviour is dependent upon the weather.

What matters, however, is not so much the particular way we express this dependence or connection, but that we recognize it as basic and undefinable in our model of meaning. By underlining this we make explicit something we implicitly learn rather late in life, when, after some difficulties, we master the concept of conditionality, when we begin to understand how conditionals and alternations by connecting the truth values of the components are wholes that determine the possible values of these components without being a function of them, hence the name truth determinant.

It may be that this approach in the philosophy of meaning is quite different from usual views about language where the meaning of composite linguistic units is treated as a function of their parts, a principle often referred to as Frege's principle. And it may be that the notion of a truth-determinantal conditional will therefore resist attempts of accounting for it by help of the traditional tools of set theory and formal semantics that presuppose Frege's principle, at least it is an open problem how to do it. But this is hardly sufficient to justify an accusation of unclarity of the concept of a truth determinant.

However, this concept, unclear or not, is helpful in understanding how many kinds of dependence or connection we may come across in propositional logic. In the words of Enderton "there are sixteen binary connectives, but only ... ten ... are really binary".[10] Out of these ten, four will have only one combination of truth values of the components under which the compound made by the connectives in question is true; so these compounds are truth determinants behaving like truth functions, in fact, all conjunctive in character. The remaning six connectives with two or more "truth" combinations will all result in genuine truth determinants; they are, using the usual truth-functional symbols, the following : ‘p ⊃ q’ (conditional), ‘p v q’ (alternation), ‘p/q’, ‘q ⊃ p’ (reversed conditional), ‘p ≡ q’ (biconditional) and ‘p ’ q’ (exclusive alternation). Incidentally, it is interesting to note how these compounds, if logically true, will illustrate "the six logical relationships between two propositions which are recognized in traditional logic", viz., in the same order, implication, subcontrarity, contrarity, subimplication, equivalence and contradictoriness. And to note that if none of these relationships holds, we say that the propositions are *independent.*[11] The illustrations clearly show how dependences may have a logical interpretation as well as a physical.

Before ending this section by trying to see how the theory of truth determinants covers concrete examples of use of language, one theoretical question remains. I claim that the theory is general and not untolerably unclear. But to what extent is it new ?

To my knowledge Reichenbach is the one who came closest to formulate a similar theory. C. I. Lewis, to be sure, distinguished clearly between what he called an intensional and extensional (truth-functional) use of 'or' and 'if-then' in a very early paper,[12] but without any general theory. Besides it seems that he abandoned the idea later on. But Reichenbach[13] developed a distinction between what he calls an adjunctive (truth-functional) interpretation of the connectives and a connective interpretation to which I am in considerable debt. A superficial reading of Reichenbach could perhaps even leave one with the impression that his distinction is the same as mine between truth functions and truth determinants. There are two reasons, however, why this is not so.

First, Reichenbach gives the following schematic definition of 'or' and argues, quite as was done above, that if we interpret 'or' in its connective sense we define a connective operation, and if in its adjunctive sense, an adjunctive operation. But by adding that the alternation is false if and only if both its terms are false, Reichenbach reduces his definition to a simple adjunctive or truth functional definition :

	'a' is true and 'b' is true	
'a v b' is true $=_{df}$	or 'a' is true and 'b' is false	(1)
	or 'a' is false and 'b' is true	
'a v b' is false $=_{df}$	'a' is false and 'b' is false	(2)

For if we accept (2) a few steps the validity of which nobody would doubt will force us back to the truth-functional interpretation of 'or'. Is, by definition, 'a' false and 'b' false if 'a v b' is false, then, by contraposition, if it is not the case that both 'a' is false and 'b' is false, then 'a v b' is true. But if 'a' is true, then it is not the case that both 'a' is false and 'b' is false. So, by transitivity, if 'a' is true, then 'a v b' is true which it cannot be unless 'v' is interpreted truth-functionally. This shows that if we want a new connective interpretation of 'or', we must admit that the falsity of a connective alternation could be due to the failure of this connection and not necessarily mean that both terms were false. It also shows the futility of all attempts[14] of claiming that a reading of the truth tables from right to left is in full conformity with ordinary usage. For if such a reading includes the cases where the compound is false, we can always prove that then we must accept the reading from left to right as well, with respect to all lines of the truth table, and are accordingly back at the truth-functional interpretation.

Second, and perhaps as a consequence of the mistake just exposed, Reichenbach does not recognize that a proper connective interpretation of 'or' will lead to another propositional logic; such an interpretation is not compatible with a number of traditional logical laws, e.g. '$p \supset (p \vee q)$', as we shall have occasion to argue in the next section.

Meanwhile let us see how very well the truth-determinantal theory of meaning is in accordance with actual use of language, in daily life and in the sciences, including mathematics it is claimed though I cannot argue this point here.

Take, to begin with, a simple example such as "if we do not build a new dam, the town will be flooded" where we state that there is some real connection between events in the empirical world. Evidently, we cannot be sure of the truth of the conditional, even if we do not build the dam and the town is flooded, for this could be for other reasons than the lacking dam. Nor can we be sure of its truth if we do build the dam and the town is flooded, since we know nothing about what would have happened if we had not built the dam. For the same reason we are no better off with respect to the truth of the conditional if we build the dam and the town is not flooded. So the conditional is not a truth function. It is false, however, if we do not build the dam and the town is still not flooded, but that it is a point where truth functions and truth determinants are alike.

So what remains in order to be convinced of the truth determinantal character of the conditional is that a connection between the three cases mentioned above is necessary and sufficient for the truth of the conditional.

That it is necessary is obvious. We would not admit the truth of the conditional, were it not true that all three possibilities were open and related in such a way that if the first did not take place then the second or the third would.

It is a little more difficult to see that establishment of such a relationship between the three possibilities is also sufficient for the truth of the conditional. However, what we are sure of in expressing "if we do not build a new dam, the town will be flooded" is that there is no way of preventing the flood without the dam. But this follows if we can show either we do not build the dam and have the flood, or we build it and have the flood or not, in the strong sense of 'or' where it expresses a connection between the possibilities.

A similar analysis seems possible if we examine an 'if-then' used for expressing a logical relationship, say, in order to connect premisses and conclusion in a valid syllogistic argument. Witness in particular how the tradition teaches that a valid syllogism may have true premisses and true conclusion, false premisses and true conclusion, and false premisses and false conclusion, whereas it is excluded that we have true premisses and false conclusion. So any attempt of tampering with the truth table or regarding the last two lines, where the antecedent of the conditional is false, as more arbitrary than the first line, would be in conflict with the most respectable part of traditional logic teaching.

Examples of conjunction and negation will not involve us in any difficulties since these truth-determinantal connectives logically behave exactly like their truth-functional relatives that are easily exemplified. But the connective 'or' for alternations remains; examples, however, should not be hard to find since any asserted alternation one could think of would probably be a truth-determinant.

So the important question now is if there are truth-functional alternations at all. There are, however. If we say "if he is a soldier or a student he will get in free" there is obviously no connection between being a soldier or a student. But the remarkable thing about this alternation is that it is not asserted; it is a pure assumption or hypothetically stated condition and in such a context it might be useful to say something about possible truth values though they are not connected in the strong sense we have introduced above. Even 'if-then' may be truth-functionally used in such contexts; though slightly clumsy, the following still seems to be idiomatic : "in case if he is not a soldier, then he is a student, he will get in free".

However, apart from such contextually used 'or's and 'if's I doubt if anybody could come up with a clear instance of a truth-functional 'or' or 'if-then'. If we consider the truth conditions for an asserted alternation or conditional, these conditions make it extremely unlikely that truth-functional assertions of this kind could be useful at all. For in order to assert a proposition we must know not only its truth conditions, but also that these conditions are actually fulfilled. Since the sufficiency condition for both alternations and conditionals is that one or another of the three listed possibilities will take place, we must have established that one actually does take place in order to assert the compound proposition. But, then, unless we are going to stress a connection and assert rather a truth determinant (e.g. when we observe that it is raining and the street is wet and argue that this is so because if it is raining, the street is wet), it is wholly pointless to assert, say, "if France is in Europe then the sea is salt" just because we know that France is in Europe and the sea is salt. This argument is *not* an argument against truth-functional logic; as we shall see in section 5 there are reasons in favour of this logic and its stipulation to take alternations and conditionals as truth-functions. It is merely an argument to the effect that a truth-functional model of meaning for these compounds, indeed for all compounds with only two components, apart from conjunction, makes it pointless ever to utter such compounds. I add without argumentation (which would be easy) that examples such as 'if $2 \times 2 = 4$, I will be there" are no counterexamples.

On the other hand, the very same truth-functional model is fully adequate for understanding how truth-functional alternations occur and are useful in context as in the example above, though often the context will be implicit or presupposed and not expressed in words.

5. *Truth-determinantal logic.*

Having introduced the concept of a truth determinant and distinguished it from a truth function, we must now ask if this concept will require a new logic. It is clear by now, I presume, that the answer is an unreserved yes. And this answer was to be expected.

If logic as Tarski said is a study of the meaning and structure of the propositional connectives, and the truth conditions and thus the meaning of a truth-determinantal conditional is different from a truth-functional one, it is not to be expected that both conditionals were the same with respect to their logical properties, with respect to what they implied or were implied by. The difference in meaning between ' $\rightarrow$ ' and ' $\supset$ ' will result in different logical behaviour. Indeed, often the logical behaviour may be the best cue for determining the meaning. For instance, if we know that if he is a soldier or a student, he will get in free and discover that he has paid, we infer that he is neither a soldier nor a student, and have no doubt that we have a truth-functional 'or' before us. If, on the other hand, we do not draw a similar conclusion by denying that (by agreement) Peter or Paul will come to-morrow (since it is Peter or Hans), it is apparent that we take the latter 'or' as a truth-determinantal 'or'. That closely knit together is logic and theory of meaning.

But there is hardly any other possibility here than to start, as propositional logic did with the Stoics, by giving a number of examples, and here of truth-determinantally valid arguments. In picking out these we should be guided by our theory of meaning if it is worth anything. Immediately we know by considering the definition of a truth-determinant that this concept will be reflected in a system, if any, that is a proper part of truth-functional logic.

Thus it seems that we may go about our task by singling out logical laws from the truth-functional system that will be valid according to our definition and at the same time rejecting others as essentially truth-functional.

By calling a law essentially truth-functional I mean that it holds only when taking the alternations and conditionals involved as truth functions. These laws, since being only true for 'v' and ' $\supset$ ', will accordingly turn these connectives into proper truth-functional connectives. Some can be got directly from simply reading the truth tables in the way only a truth-functional interpretation will permit, e.g. '$(p \,.\, q) \rightarrow (p \supset q)$' or '$(\sim p \,.\, q) \rightarrow (p \vee q)$'. (The arrow, instead of the horseshoe, is due to the fact that we have here a real logical dependence.) Others, like '$p \rightarrow (p \vee q)$' or '$p \rightarrow (q \supset p)$', by reflection on the fact that a truth-functional alternation is true as long as one of its terms is, and a conditional as long as the consequent is true. Or by realizing that the law in question, by acceptable non-truth-functional steps, will lead to an essentially truth-functional law. Thus '$((p \,.\, q) \supset r) \rightarrow (p \supset (q \supset r))$' will, by substituting 'p' for 'r' and using *modus ponens* to '$(p \,.\, q) \supset p$' and the substitution instance, lead to '$p \supset (q \supset p)$'.

On the other hand, such a law as '$(p \,.\, \sim q) \rightarrow \sim(p \rightarrow q)$' is truth-determinantally valid since an excluded case will always lead to the falsity of the conditional expressing dependence or connection. To this I add a number of other laws, none picked out *ad hoc*, but all similarly valid for ' $\rightarrow$ ' and ' $\stackrel{\vee}{=}$ ' where these signs express dependence or connection between the components in accordance with the concept of a truth determinant. Further some immediately acceptable laws concerning conjunction.

Truth-functional laws.

$(p \,.\, q) \rightarrow (p \supset q)$
$(\sim p \,.\, q) \rightarrow (p \vee q)$
$p \rightarrow (p \vee q)$
$p \rightarrow (q \supset p)$
$p \rightarrow (\sim p \supset q)$
$((p \,.\, q) \supset r) \rightarrow (p \supset (q \supset r))$
$\sim(p \supset q) \rightarrow (p \,.\, \sim q)$
$\sim(p \vee q) \rightarrow (\sim p \,.\, \sim q)$
$\sim(p \,.\, q) \rightarrow (\sim p \vee \sim q)$

Truth-determinantal laws

$(p \,.\, \sim q) \rightarrow \sim(p \rightarrow q)$
$(p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$
$(p \rightarrow \sim p) \rightarrow \sim p$
$p \rightarrow p$
$(p \Leftrightarrow q) \Leftrightarrow ((p \rightarrow q) \,.\, (q \rightarrow p))$
$(p \stackrel{\vee}{=} q) \Leftrightarrow (\sim p \rightarrow q)$
$(\sim p \stackrel{\vee}{=} q) \Leftrightarrow (p \rightarrow q)$
$(p \stackrel{\vee}{=} q) \Leftrightarrow (q \stackrel{\vee}{=} p)$
$(p \,.\, q) \Leftrightarrow (q \,.\, p)$
$(p \,.\, q) \rightarrow p$
$((p \rightarrow q) \,.\, (p \rightarrow r)) \Leftrightarrow (p \rightarrow (q \,.\, r))$

If we now treat the truth-determinantal laws stated as an axiom system, with *modus ponens* and substitution as rules of inference, not bothering about the question of independence that after all is only a matter of elegance, it seems that we have succeeded in separating the concept of connection or dependence from the essentially truth-functional laws that are, in a sense, foreign to this concept. For it can be proved[15] that if we substitute '$\rightarrow$' for '$\supset$' and '$\veebar$' for 'v' in the truth-functional laws none of these can be deduced from our set of truth-determinantal laws, or, in other words the truth-determinantal system will not lead to the full propositional calculus.

Nevertheless the truth-determinantal system, if it should be allowed the name of system, is not powerless. Quite a number of important logical laws such as all so-called hypothetical or disjunctive "syllogisms", dilemmas, indirect proofs, contrapositions, de Morgan laws (in one direction), double negation, etc. can be derived within truth-determinantal logic. Further, as other subsystems of traditional propositional logic it contains at the same time this logic, since all non-provable truth-functional laws will be truth-determinantally provable when translated into formulas with only negations and conjunctions[16].

However, there are three different kinds of consequences about what is *not* deducible that need our attention.

The first one is neither unwelcome nor quite unexpected. The so-called paradoxes '$(p \,.\, \sim p) \rightarrow q$' and '$p \rightarrow (q \veebar \sim q)$' are not truth-determinantally derivable. But this is all for the better since it is foreign to a truth-determinantal 'if-then' to have as an instance, "if it is raining and not raining, then the moon is made of green cheese."

The second consequence is not unexpected, but nonetheless troublesome. The formula '$((p \veebar q) \rightarrow r) \rightarrow ((p \rightarrow r) \,.\, (q \rightarrow r))$' is not derivable. But nor should it be since it expresses that when a connection is a sufficient condition for 'r', both terms of this connection also are which need not be true. However, we still must have a law to cover a truth-functional 'or' in such a context, where it actually does occur as we have previously seen. And if we try to include something like '$((p \vee q) \rightarrow r) \rightarrow ((p \rightarrow r) \,.\, (q \rightarrow r))$' this is essentially truth-functionally, we only have to substitute '$p \vee q$' for 'r' to have the full truth-functional calculus.

The third consequence, however, is the most serious. The so-called antilogism '$((p \,.\, q) \rightarrow r) \rightarrow ((p \,.\, \sim r) \rightarrow \sim q)$' that we need in the theory of syllogism and '$((p \,.\, \sim q) \rightarrow (r \,.\, \sim r)) \rightarrow (p \rightarrow q)$' that we use in proof procedures are not derivable, even if apparently not essentially truth-functional. In their unrestricted form, however, these laws could lead to paradox as is easily seen. Nonetheless we cannot do without the two types of reasoning so something must be done if truth-determinantal logic should not plead guilty of a serious incompleteness.

Analysis shows, however, that the paradox '$(p \,.\, \sim p) \rightarrow q$' will follow only if we substitute 'p' for 'r' in the antilogism and thus let 'r' be implied by one of the conjuncts in the conjunction '$p \,.\, q$' alone. And that is not, in fact, what we want to express by the antilogism. Rather it is 'p' and 'q' taken together that imply 'r' which is a stronger statement. So it seems that we need, particularly for use in antecedents of conditionals, a weaker conjunction than the truth-functional one. Possibly, but this is no more than a suggestion, we could define this conjunction '$p \circ q =_{df} \sim (p \rightarrow \sim q)$'. At least this definition would make the two laws wanted provable in truth-determinantal logic, the antilogism, to be sure, only in the form '$((p \circ q) \rightarrow r) \rightarrow ((p \,.\, \sim r) \rightarrow \sim q)$' where the consequent is weaker than its corresponding antecedent due to the different conjunctions. But it is not obvious why we should need a stronger principle.

Further, with a non-truth-functional 'and' we could also prove *modus ponendo tollens* that fails and should fail for a truth-functional 'and'; that is, '$(p \,.\, \sim(p \circ q)) \rightarrow \sim q$' is truth-determinantally valid whereas '$(p \,.\, \sim(p \,.\, q)) \rightarrow \sim q$' is not. Witness in this connection how '$\sim p \rightarrow \sim(p \,.\, q)$' is a truth of logic in our system, but '$\sim p \rightarrow \sim(p \circ q)$' is not.

5. *Traditional mathematical logic, its paradoxes and its UNSOUNDNESS.*

When a possible truth-determinantal logic has to face the difficulties mentioned in the last section, one may wonder why it would not be the best thing just to keep truth-functional logic as we know it and as it is taught all over the world.

The reason is not the paradoxes or awkward propositions of this logic such as "if France is in Europe then the sea is salt". For there is a very powerful defence of an extension of language that has such results, viz. that it leads to a smooth-running and complete systematic logic. With such a scientific purpose in mind, it is argued quite correctly, it is fruitful to take 'if-then' and 'or' in their weakest sense, as it were, to find their least common denominator, provided we keep sufficiently sense to be able to account for the role 'or' and 'if-then' have in inferences. This actually seems to be the case since all valid propositional arguments are cared for in mathematical logic. Further, it is argued, even as correctly, that by a proposition such as 'if France is in Europe, then the sea is salt'' we are never in trouble, since it cannot be used for inferring false propositions from true ones. And the moment we realize that in this example 'if-then' is actually truth-functionally used, all our qualms should be gone.

These forms of defence, however, will be of no avail when, enlightened by the truth-determinantal theory of meaning, we show that mathematical logic is unsound or illogical when applied. For it generally permits a conclusion from a denial of an asserted conditional or alternation to the truth value of the components of these compounds. So mathematical logic, if it recognizes only truth-functional compounds, is much too strong.

Indeed, to have as a logical law that from denying an asserted conditional you may infer the truth of the antecedent and the falsity of the consequent is little better than having a law saying that from affirming the consequent you could infer the truth of the antecedent of a conditional. Both inferences may turn out to lead to true, but as often to false assertions, and you would never claim validity for such inferences. None of them, happily, is licensed by the truth-determinantal system, and it is difficult to tell why a system that justifies the former inference, like mathematical logic does, has been tolerated. After all soundness must be prior to completeness. So it is something of a scandal to philosophy and mathematics that truth-functional logic is taught as beginning logic to innocent students.

The main reason probably is that logicians have been such impressed by the formal beauty of the truth-functional, complete logic that they have put the blind eye to its applications where these will lead to difficulties, helped by the fact that the fallacious inferences in question are not likely to occur in actual reasoning. Some logicians, however, have seen these difficulties without being seriously troubled by them. In Benson Mates' textbook[17] there is a clear sign on page 76 that he is aware that mathematical logic may be too strong; this, however, does not prevent him from setting an excersize (pp. 104-105) as the following : "For each of the following arguments try to symbolize premisses and conclusion by means of SC sentences and then derive the symbolized conclusion from the symbolized premisses : d) If Henry gets a Rolls-Royce for Christmas, he will have to park it in the street unless someone gives him a garage, too. It is not true that if he is a good boy then someone will give him a garage. But if Henry is a good boy he will get a Rolls-Royce for Christmas. Therefore, he will indeed get a Rolls-Royce for Christmas and will have to park it in the street." Here, by denying a conditional, you get a conclusion that quite obviously does not follow from its premisses. To be sure, in his preface (p. VI) Mates mentions the possibility of such results when applying mathematical logic; but this seems to be like setting out to draw a map and at the same time warn against its use, since it may lead you in the wrong direction.

If it should be argued against my criticism of mathematical logic that this logic creates its own formalized language that cannot be judged by its relations to the imperfect natural language, the two following points may be argued :

1) Either you must allow application of terms from the formalized language to natural language or not; you cannot apply the horseshoe '$\supset$' to some arguments involving conditionals, say, dilemmas and hypothetical inferences, and find them validated, and forbid the application when, perhaps, the very same conditional is denied, simply because there you go wrong.

2) In case you deny altogether an application of the formalized language and think of this rather as a perfected substitute language, you cannot propose such a language that lacks means for expressing the important concept of connection, dependence, conditionality or how you wish to phrase it. And if it is suggested that the sign '$\supset$' does the job, a scrutiny of mathematical logic will show that it does it much too well, by leading to a number of essentially truth-functional laws that are not characteristic of the concept of dependence. However, when we compare truth-functional logic with truth-determinantal logic, the latter stands out as a system proposed to catch exactly the properties of alternations and conditionals when these compounds express a connection between their components.

The upshot of this discussion is that truth-functional logic is not acceptable as the only logic, not because of its inherent paradoxes, but simply because it brings us in the worst situation of all : we may deduce false conclusions from true premisses. Returning to a truth-determinantal logic would not do either, even if the suggested amendments for the sake of the antilogism and other necessary laws that do not seem to be essentially truth-functional, were acceptable. For there are genuine truth functions in language, at least in context. And if we want to cover these by a general logic, the whole truth-functional system is bound to follow.

The way out seems to be a combination of a system of truth determinants with the traditional truth-functional system. Our formal tools, then, would allow us to apply logic to 'or's and 'if's with both meanings, as long as we made sure beforehand which kind of meaning was in question. At the same time we could explain the paradoxes and awkward propositions of mathematical logic as due to a generalisation of a truth-functional sense the connectives do have in language, but only in context.

This last remark naturally suggests a comparison of the ideas behind the attempts of working out a truth-determinantal logic with a much more elaborate, but related logic already at hand, the Anderson & Belnap system or rather systems of relevance logic[18]. Anderson & Belnap are certainly not among the logicians that have not been troubled by the difficulties mentioned above pp. 62-63. On the contrary, they have forceful examples showing how we may deduce a false proposition from a true one by correctly denying a conditional, in particular a logical conditional. But neither their criticism nor their suggested revision of logic seem to be sufficiently grounded upon a general, philosophically justified, theory of meaning. So the fact that we are often as bad off when denying alternations escapes them.

As it is, the main difference between Anderson & Belnap's and the truth-determinantal approach turns exactly on the analysis of alternations. They are prepared to accept '$p \rightarrow (p \vee q)$' as a logical law. To me this law by being essentially truth-functional not only requires a truth-functional alternation that may be asserted, without being dependent upon any conditions. It is also a law that by appending the wholly irrelevant 'q' necessarily brings paradox with it, as is evident in the so-called independent Lewis-proof of the paradox '$(p \,.\, {\sim}p) \rightarrow q$'. And this paradox could be tolerated and explained as due to a generalisation of meaning for the purpose of a scientific systematisation, but not as something that is inherent in language and deducible by a natural 'or'.

However, since Anderson & Belnap have '$p \rightarrow (p \vee q)$' and nonetheless want to avoid '$(p \,.\, {\sim}p) \rightarrow q$' altogether, they cannot strike at the root of this paradox, but must propose a remedy that does the job but seems rather ad hoc. They reject the disjunctive syllogism. To be sure, they are willing to recognize the validity of this syllogism for a stronger meaning of 'or' which they recognize but do not systematise. That is to say that the disjunctive syllogism is not immediately covered for in their logic, another serious incompleteness besides the antilogism and other similar laws also lacking in the Anderson & Belnap system. Further, even their rejection of the disjunctive syllogism for the weaker 'or' is not consonant, it seems, with the general truth-functional theory of meaning for the 'v' sign of alternation that certainly turns '$({\sim}p \,.\, (p \vee q)) \rightarrow q$' into a valid law.

The conclusion that forces itself upon us, if the arguments of this paper are correct, is that we are in the dark about propositional logic.

We cannot any longer be content with a truth-functional logic by which false conclusions follow from true premisses; on the other hand, by way of systematic logic we have as yet no satisfactory and complete system to offer. So there is room for suggestions, and one such suggestion is the system of truth-determinants above combined with ordinary truth-functional logic. Only further study of this system and its properties will tell if it contains a more adequate logic than the traditional one. At least it seems a bit too early to give up all hopes of a fully systematic logic and return to the Stoics, gathering examples of valid reasoning in our herbarium, and remain satisfied with rather limited unifying principles and no general models of meaning.

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NOTES

[1] See his *Frege's Philosophy of Language*, (London 1973), in particular chapter 19.

[2] op. cit., p. 668.

[3] Alfred Tarski, *Introduction to Logic*, (New York 1946), p. 19.

[4] I have done so in *On the Nature of Meanings*, (Copenhagen 1965).

[5] See Jorgen Jorgensen, "Reflexions on Logic and Language", *Journal of Unified Science*, vol. VIII (1939), p. 223.

[6] W.V. Quine, *Methods of Logic*, (London 1974), p. 15.

[7] From Tarski's and Quine's books referred to above.

[8] "On the History of the Logic of Propositions" in *Jan Lukasiewicz. Selected Works*, (Amsterdam 1970), p. 207.

[9] An English translation in van Heijenoort *From Frege to Gödel*, (Cambridge, Mass. 1967).

[10] *A Mathematical Introduction to Logic*, (New York 1972), p. 50.

[11] See E.J. Lemmon, *Beginning Logic*, (London 1965), pp. 69-70.

[12] See his "Implication and the Algebra of Logic" *Mind*, vol. XXI (1912), reprinted in *Collected Papers of Clarence Irving Lewis*, (Stanford 1970).

[13] *Elements of Symbolic Logic*, (New York 1947), pp. 27-34.

[14] There is such an attempt with Scholz & Hasenjaeger *Grundzüge der mathematischen Logik*, (Berlin 1961), pp. 31-32.

[15] By use of some three-valued matrics. See my "Is there a Logic or Formal System Based on the Concept of a Truth Determinant ? ", *Danish Yearbook of Philosophy*, vol. X (1973), pp. 81-82.

[16] This is easily established by proving the laws corresponding to the three axiom schemes Barkley Rosser has proved formally complete for a system containing only '~' and '.'. See his *Logic for Mathematicians*, (New York 1953), pp. 73-74).

[17] ***Elementary Logic***, (New York 1965).

[18] Now most profitably studied in their *Entailment I*, (Princeton 1975).